International Trade
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International Trade (8402)

Fall 2007, Mini 2

Problem set 2

Due Wednesday, Nov. 30, in class.

Consider the standard symmetric 1 good international real business cycle model where households in both countries can trade in stocks.

Domestic households objective

$$\max E \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1-\sigma} c_t^{1-\sigma} - \phi \frac{l_t^v}{v} \right)$$

Foreign households are exactly symmetric

Domestic firms objective

$$\begin{aligned} & \max E Q_t d_t \\ d_t &= A_t k_{t-1}^{\alpha} l_t^{1-\alpha} - w_t l_t - x_t \\ k_t &= (1-\delta) k_t + x_t - \eta k_{t-1} \left(\frac{x_t}{k_{t-1}} - \delta \right)^2 \end{aligned}$$

where Q_t is price uses by firms to value dividends in different states/dates. Initially assume that $Q_t = U_{c,t}$ i.e. that domestic firms care only about domestic households, regardless of the equilibrium stock holdings. Foreign firms are exactly symmetric.

Budget constraints are

$$\begin{aligned} c_t + \lambda_t^D p_t + \lambda_t^F p_t^* &= w_t l_t + \lambda_{t-1}^D (p_t + d_t) + \lambda_{t-1}^F (p_t^* + d_t^*) \\ c_t + \lambda_t^{*F} p_t^* + \lambda_t^{*D} p_t &= w_t^* l_t^* + \lambda_{t-1}^{*D} (p_t + d_t) + \lambda_{t-1}^{*F} (p_t^* + d_t^*) \end{aligned}$$

where

λ_t^D Share of domestic stock held by domestic households
λ_t^F Share of foreign stock held by domestic households
λ_t^{*D} Share of domestic stock held by foreign households
λ_t^{*F} Share of foreign stock held by foreign households

Assume that A_t and A_t^* follow independent AR(1) process with persistence parameter $\rho = 0.95$ and standard deviation of innovations equal to 0.01. Assume that $\sigma = 2, \beta = 0,99, \alpha = 0.3, \delta = 0.025, \nu = 2$ and calibrate ϕ and η so that in the complete market version of the model the average labor is 0.3 and investment is 3 times as volatile as GDP.

Compute the average equilibrium portfolio holdings of foreign assets. In particular do the following. Guess a steady state portfolio (for example $\lambda_t^D = \lambda_t^{*F} = 1, \lambda_t^F = \lambda_t^{*D} = 0$). Around this guess solve for decision rules approximated up to the second order. Note that in order to do so you will have to first impose a small cost of holding portfolio different from the steady state (i.e. impose a resource cost $\xi(\lambda_t - \lambda_0)^2$, where ξ is a small number and λ_0 is your initial guess) and then use one of the packages available. One package which works well is the one developed by Stephanie Schmitt-Grohe & Martin Uribe, in the paper "Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function" (MATLAB programs are available on their web-page but you'll need the symbolic toolbox), alternatively you can try DYNARE++.Simulate your model for a number of periods and then check whether the average stock holding in the simulation are similar to the your initial guess of the steady state. If they are not update your initial guess until convergence.

Once you obtained your decision rules do the following things:

i) Check that the portfolio solution does not change as you locally change ξ

ii) Assess how average portfolio changes as you change the persistence of the shocks. In particular graph the average share of foreign assets held in equilibrium as a function of the persistence of the shocks, with the persistence going from 0 to 1.

iii) Assess how average portfolio changes as you change the correlation of the innovation of the shocks. In particular graph the average share of foreign assets held in equilibrium as a function of the correlation of the shocks with the correlation going from 0 to 0.5

iv) Check how the solution changes when you assume that $Q_t = \lambda_{t-1}^D U_{ct} + \lambda_{t-1}^{*D} U_{ct}^*$. (assume that agents do not take into account that their decision of changing stock position affects the objective of the firm)