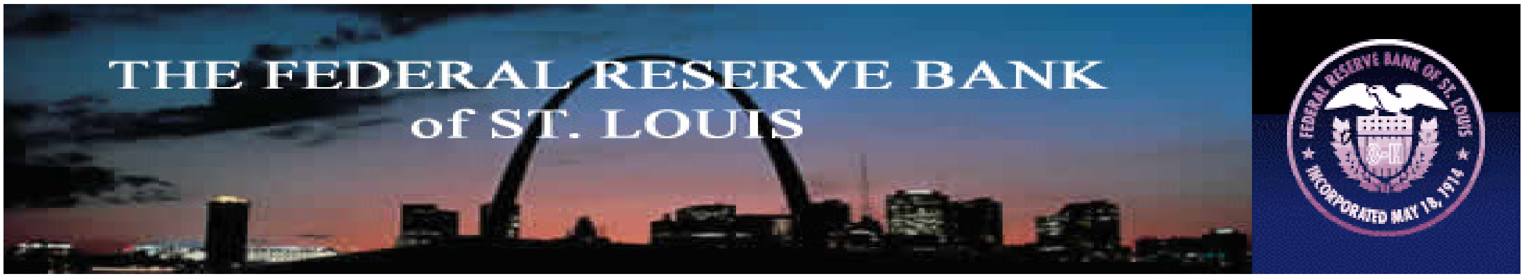

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A Spectral Analysis of the Cross-Country Consumption Correlation Puzzle

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Abstract

Dynamic general equilibrium models predict high cross-country consumption correlations, whereas the data show that output correlations tend to be higher. Spectral decomposition reveals that this ranking varies across frequency bands, with consumption correlations often exceeding output correlations at higher frequencies.

*The views expressed in this paper are those of the author, and do not necessarily represent official positions of the Federal Reserve Bank of St. Louis or the Federal Reserve System.

A Spectral Analysis of the Cross-Country Consumption Correlation Puzzle

1. The Correlation Puzzle

When trade patterns between nations are modeled as general equilibrium allocations between risk-averse trading partners, consumption-smoothing motives imply a high correlation of consumption across countries. Indeed, in a simple endowment setting with complete asset markets (in the Arrow-Debreu sense), consumption will be perfectly correlated across countries, regardless of the cross-country output correlation.

However, the data show a clear tendency for cross-country output correlations to be higher than cross-country consumption correlations. This feature is illustrated in Table 1, which shows the correlations of quarterly growth rates for output and consumption with rest-of-world variables for 10 OECD countries.[1] For each of the countries shown, the output correlation is higher than the consumption correlation. Backus, Kehoe and Kydland (1992) called this observation "the most striking discrepancy ... between theory and data," and efforts to reconcile this disparity have generated a sizable literature.[2]

This note presents a spectral decomposition of the consumption/output correlation puzzle, showing that the relative ranking of these cross-country correlations varies across frequency bands. In particular, the finding that output correlations exceed consumption correlations is shown to be prevalent only within the range of frequencies generally associated with business cycle fluctuations. At both higher and lower frequencies, cross-country consumption correlations show a greater tendency to exceed output correlations. This finding introduces a new dimension to the correlation puzzle.

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[1]Data are from the OECD Quarterly National Accounts, using a sample period of 1973:Q1 to 2002:Q4. Rest of world variables are constructed by subtracting real consumption and output for each country from the OECD totals (expressed in 1996 U.S. dollars).

[2]Surveys of this literature are included in Backus, Kehoe and Kydland (1995) and Baxter (1995). More recent examples include Maffezzoli (2000), Baxter and Farr (2001), Heathcote and Perri (2002), and Kehoe and Perri (2002).

2. Spectral Decomposition

Correlations like those shown in Figure 1 summarize the comovement of two variables over a full range of frequencies. The benefit of applying a spectral approach to the analysis is that it allows for the decomposition of these moments into constituent frequency components—providing a richer representation of dynamic interactions.

The spectral representation theorem states that any real valued, covariance stationary process can be represented as the weighted sum of orthogonal periodic components. Specifically, the spectrum of a vector of time series can be expressed in terms of a Fourier sum:

$$s(\omega) = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty}\Gamma_k e^{-i\omega k} \tag{1}$$

where Γ_k is the kth-order autocovariance matrix of the vector of time series. Note that the spectrum incorporates all information about the variance-covariance structure. In fact, the autocovariance matrix can be recovered from the spectrum using the Fourier integral:

$$\Gamma_k = \int_{-\pi}^{\pi} e^{i\omega k} s(\omega) d\omega. \tag{2}$$

For the special case of k=0:

$$\Gamma_0 = \int_{-\pi}^{\pi} s(\omega) d\omega, \tag{3}$$

that is, the spectrum integrates to the contemporaneous cross-covariance matrix. The real-valued components of the off-diagonal elements of the spectrum, known as the *cospectra*, integrate to the covariance between two series. Hence, a plot of the cospectrum for a pair of series provides a frequency-domain decomposition of their correlation.

For a sample of T observations, the spectra and cospectra of a vector of time series can be estimated using a discrete version of equation (2), using a set of frequencies $\omega_j = 2\pi j/T$, j=1, ...T/2. In the interest of obtaining consistent estimates, it is common to estimate weighted averages of the raw spectral densities, or equivalently, to use a lag window approach which both weights and limits the number of autocovariances used:

$$\hat{s}(\omega) = \frac{1}{2\pi} \sum_{k=-(T-1)}^{(T-1)} w(k)\hat{\Gamma}_k e^{-i\omega k} \tag{4}$$

where the kernel *w(k)* is a matrix of lag windows. This paper employs the Blackman-Tukey kernel using

$$w(k) = \begin{cases} [1+\cos(\pi k/h)/2 & for \quad |k| \leq h \\ 0 & otherwise \end{cases}$$

with the lag truncation parameter $h = 2\sqrt{T}$.

Figure 1 shows the sample cospectra for consumption and output growth rates, normalized so that each integrates to the corresponding correlation shown in Table 1. Within the range of frequencies generally associated with business cycles–approximately 6 to 32 quarters in periodicity (π/3 to π/16 in frequency)–the tendency of the output correlation to exceed the consumption correlation clearly predominates. Both the consumption and output correlations have much of their weight concentrated within this frequency band. A second, higher frequency peak is also evident in most of the cospectra, but for this peak the consumption cospectrum often exceeds the output cospectrum. At frequencies below the business cycle range, the consumption cospectrum is higher in some cases as well.

Table 2 shows the breakdown by frequency bands of the correlations from Table 1, calculated by integrating the cospectra in Figure 1. The consumption correlation puzzle is most pronounced in the business cycle frequency range. However, the higher-frequency components of the consumption correlations are higher than corresponding output components for eight of the ten countries. For frequencies below the business cycle range, the consumption correlation component is higher for four countries. Associating a low consumption correlation with a lack of international risk-sharing, the decompositions in Figure 1 and Table 2 suggest that the “puzzle” is prevalent only at business cycle frequencies, with evidence of risk-sharing being more apparent outside that band, particularly at higher frequencies.

3. A Band-Pass Filter Approach

As decompositions of growth-rate correlations, the breakdowns shown in Figure 1 and Table 2 are subject to some bias due to first-differencing. The transfer function of the first-difference filter, which shows the proportion of variation in the underlying series that is "transferred" to the first-differenced series, can be expressed as $2\times[1-\cos(\omega)]$, which is greater than one for frequencies higher than the business cycle frequency band; that is, first-differencing boosts the relative importance of high-frequency components of time series.

An alternative approach to frequency decomposition of consumption and output correlations is to apply of a band-pass filter to the log-levels of the underlying series. Table 3 reports the correlations of series that have been filtered using the algorithm described by Christiano and Fitzgerald (2003).

Using the band-pass filter to isolate fluctuations within the same frequency bands presented in Table 2, the statistics in Table 3 show the same patterns. Within the business-cycle frequency band, the output correlation is higher in nine of the ten comparisons. In the high-frequency range, however, nine out of ten comparisons show the consumption correlations to be higher. For low-frequency fluctuations the patterns is mixed, with output correlations higher for four countries and consumption correlations higher for the remaining six.

Finally, as an additional check on the robustness of these results, Table 4 reports the summary of a comparison of bilateral correlations among the ten countries in the data set. As is the case with rest-of-world correlations, the consumption correlation puzzle is predominant in the business cycle frequency range. However, consumption correlations exceed output correlations in more than half of the comparisons of higher-frequency fluctuations, using either technique.

4. Conclusion

The finding of non-uniform ranking of consumption and output comovements across different frequency bands adds an additional dimension to the cross-country consumption correlation puzzle. On the one hand, the finding that relatively low consumption correlations are predominant in the business-cycle frequency range validates the focus of research on business cycle fluctuations. In particular, the finding verifies that the pattern of correlations is not simply an artifact of the Hodrick-Prescott filter that is often applied to the data.[3] Nevertheless, if we take the observation of low cross-country consumption correlations to be indicative of a lack of international risk sharing, it is curious that there is stronger evidence of risk sharing at higher frequencies than within the business cycle range.

It may simply be the case that the relative ranking of cross-country output and consumption correlations is not a robust measure of international risk sharing. Alternative characterizations, such as the domestic savings/investment correlation (Feldstein and Horioka, 1980) or the correlation between domestic consumption and world income (Lewis, 1995; Pakko, 1996) may prove to be more robust representations of the general risk sharing puzzle.

[3]Cogley and Nason (1995) have suggested that the H-P filter can induce spurious fluctuations at business-cycle frequencies.

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Table 1:
Correlations of Output and Consumption Growth with Rest of World

	Output	Consumption
Australia	0.293 *	0.174
Canada	0.517 *	0.342
France	0.530 *	0.491
Germany	0.406 *	0.399
Italy	0.401 *	0.195
Japan	0.198 *	0.147
Spain	0.207 *	0.153
Sweden	0.414 *	0.298
U.K.	0.411 *	0.348
U.S.	0.416 *	0.269

Notes: Correlations of logged first-differences of OECD *Quarterly National Accounts* data, 1973:Q1-2002:Q4. Rest-of-world variables are constructed by subtracting own-country aggregates from OECD totals. An asterisk (*) indicates the greater of the output or consumption correlation.

Table 2:
Spectral Decomposition of Output and Consumption Correlations

	Low Frequency		Business Cycle Frequency		High Frequency	
	Output	Consumption	Output	Consumption	Output	Consumption
Australia	0.047 *	0.002	0.198 *	0.002	0.048	0.169 *
Canada	0.102 *	0.101	0.354 *	0.117	0.061	0.125 *
France	0.099 *	0.092	0.282 *	0.126	0.148	0.273 *
Germany	0.032	0.085 *	0.250 *	0.077	0.124	0.237 *
Italy	0.061	0.078 *	0.289 *	0.078	0.051 *	0.039
Japan	0.052 *	0.011	0.097 *	0.031	0.049	0.104 *
Spain	0.072	0.109 *	0.103 *	0.061	0.032 *	-0.018
Sweden	0.092	0.120 *	0.309 *	0.070	0.013	0.109 *
U.K.	0.109 *	0.091	0.158 *	0.100	0.144	0.157 *
U.S.	0.081 *	0.044	0.261 *	0.092	0.074	0.134 *

Notes: See Notes to Table 1. The business cycle frequency band is specified as $\pi/16$ to $\pi/3$ (periodicity 6-32 quarters).

Table 3:
Correlations of Band-Pass Filtered Data

	Low Frequency		Business Cycle Frequency		High Frequency		Full Frequency Range	
	Output	Consumption	Output	Consumption	Output	Consumption	Output	Consumption
Australia	0.854 *	0.678	0.441 *	-0.249	0.118	0.272 *	0.622 *	0.125
Canada	0.863	0.889 *	0.589 *	0.172	0.142	0.255 *	0.764 *	0.739
France	0.801	0.846 *	0.563 *	0.427	0.416	0.443 *	0.698	0.751 *
Germany	-0.040	0.796 *	0.795 *	0.597	0.301	0.385 *	0.341	0.753 *
Italy	0.969 *	0.794	0.481 *	-0.061	0.162	0.189 *	0.701 *	0.491
Japan	0.676 *	0.520	0.334	0.356 *	0.073	0.202 *	0.558 *	0.399
Spain	0.865	0.912 *	0.111 *	-0.104	-0.031 *	-0.066	0.612	0.682 *
Sweden	0.501	0.764 *	0.629 *	0.498	0.069	0.331 *	0.524	0.641 *
U.K.	0.842	0.940 *	0.546 *	0.370	0.259	0.291 *	0.736 *	0.709
U.S.	0.686 *	0.479	0.660 *	0.616	0.071	0.285 *	0.629 *	0.455

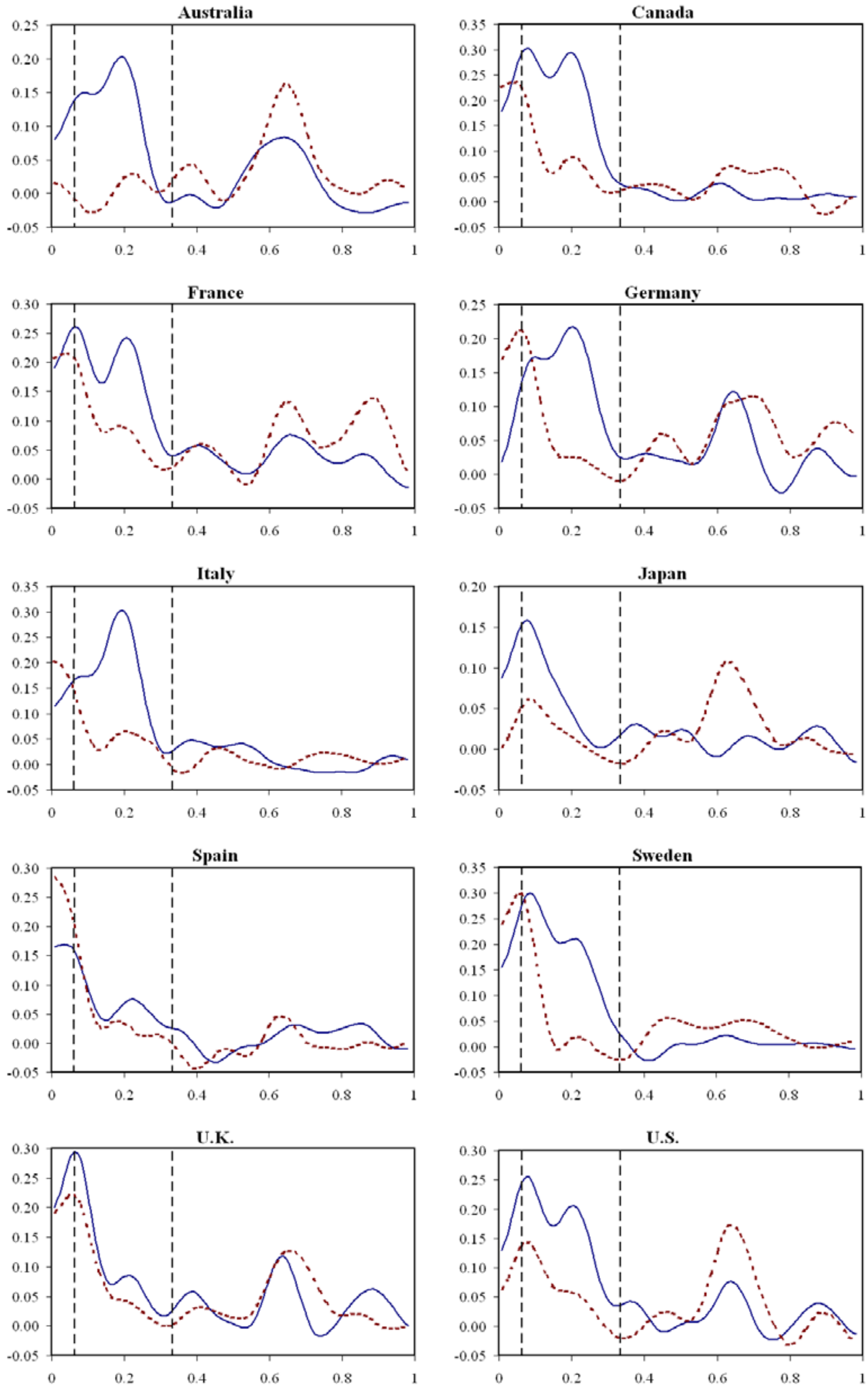
Notes: Correlations of data constructed by applying the approximate band-pass filter of Christiano and Fitzgerald (2003) to log-levels of the underlying series. Frequency Bands: High, 2-5 quarters; Business Cycle, 6-32 quarters; Low, 33-56 quarters. An asterisk (*) indicates the greater of the output or consumption correlation.

Table 4:
Rankings of Bilateral Correlations
(Number of Cases With Consumption Correlation Exceeding Output Correlation)

	Frequency Range			
	Low	Business Cycle	High	Full
Cospectrum Decomposition	18	3	26	10
Band-Pass Filtered Data	30	7	25	22

Notes: Details of data described in Tables 1-3. Each cell reports the number of country-pairs for which cross-country consumption correlation exceeds the output correlation among 10 OECD countries (a total of 45 bilateral comparisons).

Figure 1:
Cospectra of Output and Consumption with Rest-of-World Variables



Note: Horizontal axes measure frequency as a fraction of π.

— Output ···· Consumption