Commodity trade and international risk sharing

How much do financial markets matter?*

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This paper evaluates the social gains from international risk sharing in some simple general-equilibrium models with output uncertainty. A simulation model calibrated to selected moments of U.S. and Japanese data estimates the incremental loss from a ban on international portfolio diversification to be on the order of 0.20 percent of output per year. Even the theoretical gains from asset trade may disappear under alternative sets of assumptions on preferences and technology. The paper argues that the small magnitude of potential trade gains may help explain the apparently inconsistent findings of empirical studies on the degree of international capital mobility.

1. Introduction

Empirical studies of financial interdependence among industrialized economies have uncovered three major facts that together constitute a

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puzzle. The first fact is that yields on comparable assets seem to be well arbitraged across borders (absent binding official restrictions), a finding consistent with a high degree of cross-border capital mobility. The second fact, which appears to contradict the last one, is that for most major industrial countries the extent of foreign portfolio diversification is too low to be explained by standard models of financially integrated economies. A third fact, documented most prominently by Feldstein and Horioka (1980), is the generally low extent of international intertemporal trade, as measured by current-account balances. Some have argued that the small average size of current accounts, like the size of international portfolio positions, indicates high barriers to international asset trade.¹

In this paper we propose a partial reconciliation of these seemingly contradictory findings. Our basic point is that the direct welfare gains from cross-border portfolio diversification — gains from the international pooling of national consumption risks — may be quite small as far as large industrialized economies are concerned. That thesis leads to the following coherent interpretation of observed financial relationships among industrialized countries. When the gains from diversification abroad are small, even minor impediments to asset trade can wipe them out. Similarly, minor trade impediments can wipe out small gains from consumption-smoothing intertemporal exchanges. International interest-rate differentials will remain within narrow limits despite small transaction costs, and national capital markets might still be quite open to foreign linkages at the margin. A world with small barriers to foreign asset trade, but with limited gains to diversification, is therefore a world that could generate the empirical findings described above. Our formal analysis is confined for the most part to asset diversification, rather than intertemporal trade, among industrial countries; but we note in our conclusion some preliminary evidence of small potential intertemporal-trade gains within that country group.

We make our case that the gains from international risk sharing may be small by examining the theoretical and empirical implications of some completely specified general-equilibrium models of rudimentary world economies.² In particular, simulation experiments based on a calibrated endowment model with exogenous stochastic growth show that the welfare loss from prohibiting international diversification probably is on the order of

¹Some of the evidence on international financial integration is surveyed by Obstfeld (1986), Dooley, Frankel, and Mathieson (1987), and Tesar (1991). Golub (1990a) discusses the extent of international portfolio diversification by OECD countries; he shows that while diversification into foreign equities is substantial for some countries (e.g., the Netherlands, the United Kingdom, and Germany), it is quite small for many others, including the United States and Japan. [See also French and Poterba (1990).] In the past decade the current-account imbalances of some major countries reached postwar-record levels, with the result that the original regression findings of Feldstein and Horioka (1980) have been weakened; see Frankel (1991).

²This paper therefore follows the approach in Cole (1988), which develops alternative models for studying the aggregate implications of different international risk-sharing arrangements.
0.20 percent of national product per year. The models we use are, admittedly, stylized; and a litany of important qualifications to our results is given in the conclusion of this paper. Nonetheless, we take the models seriously as parables that yield important lessons and as springboards for further quantitative research. A major advantage of the general-equilibrium approach, one that in our view amply justifies the simplifications it requires, is that it gives a complete account of the mechanisms through which economic disturbances are transmitted among countries.

Given our underlying consumer-preference model and the stochastic properties of industrial-country output growth, it is not surprising that we find small gains from international risk sharing. Using a related model, Lucas (1987) estimates the cost of postwar United States consumption variability to be quite small.3

A crucial mechanism underlying our results, however, and absent from Lucas's (1987) framework, is the effect of output shocks on the relative prices at which international commodity trade occurs. We find that fluctuations in international terms of trade can play an important role in automatically pooling national output risks, since (other things equal) a country's terms of trade are negatively correlated with growth in its export sector. Indeed, the models we work with below to highlight this effect, although quite standard, have the property that for certain parameter choices terms-of-trade responses alone provide perfect insurance against output shocks. In such cases the gains from international portfolio diversification (and possibly the gains from intertemporal foreign trade as well) are nil. These are knife-edge conclusions, but our simulations suggest that for major industrial countries they may not be dramatically inaccurate.4

In section 2 we use Lucas's (1982) barter model of perfectly-pooled world financial markets to derive a first example of an economy in which international asset trade is redundant.5 The finding that world equilibrium can be Pareto-efficient even when international asset trade is prohibited is fragile.

3Unlike the model in Lucas (1987), however, ours incorporates stochastic trends in consumption. These contribute to welfare losses somewhat bigger than those Lucas finds. Another result similar in spirit to those reported below is Cochrane's (1989) estimate of minor costs due to empirically plausible departures from individually optimal intertemporal consumption plans.

4Naturally, the welfare effects of missing risk markets have received extensive attention in the literature; see, for example, Stiglitz (1982), Newbery and Stiglitz (1982), and the references therein for relevant results. Our theoretical investigation can be viewed as an extension, based on a different class of models, of the program pursued by these authors. Cobb–Douglas preferences, with their implication of unitary price elasticities, play a special role in all these models. Helpman and Razin (1978), in an international context, earlier noted the special implications of Cobb–Douglas preferences for the substitutability of equities in different industries. Newbery and Stiglitz (1982) present some approximate welfare-cost calculations whose message is the same as that of the simulations in section 4 below: for empirically plausible cases, the efficiency losses caused by missing risk markets may be small.

5By calling asset trade 'redundant', we will mean that the allocation reached without asset trade cannot be Pareto-improved by introducing asset markets and making lump-sum transfers.
We illustrate this fragility by discussing the role of alternative assumptions about both consumer preferences and the international distribution and tradability of output endowments.

Lucas's economy is a pure exchange economy; section 2's results are also extremely sensitive to this, as extensions of the model that incorporate investment show. In section 3 we present an investment model in which, again under very special assumptions, the earlier asset-market redundancy result carries through. (Readers can skip section 3 without serious loss of continuity.)

Section 4 contains the paper's numerical results. In the simulations we conduct, which draw on U.S. and Japanese data for some key parameter values, preferences are varied to allow for various degrees of risk aversion and various elasticities of intratemporal substitution between national outputs. (Included is an infinite elasticity for the perfect-substitution case.) Under all plausible parameter combinations, the gains from international portfolio diversification are measured to be quite small.

Section 5 summarizes and points to future research directions.

2. A pure exchange economy

This section analyzes a two-country pure exchange economy under two polar assumptions: perfect integration of international asset markets and complete absence of international asset trade. The main conclusion is that under some often-assumed but restrictive conditions, the portfolio-autarky equilibrium results in a Pareto-optimal allocation. Since any Pareto-optimal allocation corresponds to the competitive equilibrium of an economy with complete, integrated asset markets, financial integration has no observable implications in these examples.

All the models we use assume a representative resident within each country. This abstraction does not imply a belief that onshore financial markets literally are perfect. Rather, our goal is to evaluate the incremental welfare gain that international diversification opportunities offer. A finding that these incremental gains are small does not imply that financial markets in general are unimportant. It does imply that the potential reduction in individual consumption variability available from purely domestic diversification opportunities leaves little scope for further reduction through additional diversification abroad.

2.1. Equilibrium with specialized endowments

The basic model comes from Lucas (1982). Two countries, the 'home' and 'foreign' countries, have stochastic endowments of distinct national outputs,
denoted by $X$ and $Y$. Home-country residents maximize

$$U_0 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(x_t, y_t) \right\}, \quad \beta < 1,$$

where $x_t$ and $y_t$ are their consumptions of the home and foreign outputs. Foreign-country residents maximize the same function of their own consumptions, $x_t^*$ and $y_t^*$. There are no international transport costs.

Lucas (1982) studied a perfectly-pooled equilibrium of the model in which a representative world agent owns half of each country's endowment process. Equilibria of the model with representative national agents can be generated, however, by considering a global central planning problem in which command allocations maximize a social welfare function. Solutions to this problem are Pareto optima whose counterpart equilibria generally involve differing national wealth and consumption levels.

Here, because of the intertemporal separability of preferences, the planning problem takes the (static) form:

$$\text{maximize} \quad \mu \cdot u(x, y) + (1 - \mu) \cdot u(x^*, y^*),$$

subject to the resource constraints

$$x + x^* = X, \quad y + y^* = Y. \quad (1)$$

Above, $\mu$ is a planner weight that determines relative national wealth levels in the counterpart market equilibrium. Optimal allocations are determined by (1) and the first-order conditions

$$u_j(x, y)/u_j(x^*, y^*) = (1 - \mu)/\mu, \quad j = x, y. \quad (2)$$

This fundamental condition states that the international ratio of marginal utilities from consuming any good must be constant across goods and across states of nature. In statistical terms, condition (2) implies that national marginal utilities from consuming any good are perfectly positively correlated. We use condition (2) repeatedly below to test whether allocations are Pareto-optimal.

Specialize now to the Cobb-Douglas/isoelastic-preference case, $u(x, y) = (x^\theta y^{1-\theta})^{1-R}/(1-R), \quad R > 0$. The planning solution is

$$x = \omega X, \quad x^* = (1 - \omega) X, \quad (3)$$

$$y = \omega Y, \quad y^* = (1 - \omega) Y,$$

where, defining $\sigma = 1/R$,

$$\omega = 1/[1 + [(1 - \mu)/\mu]^{\sigma}]. \quad (4)$$
Only when $\mu = \frac{1}{2}$ are national wealths equal—the Lucas (1982) case ($\omega = \frac{1}{2} = 1 - \omega$). Different $\mu$ values, however, imply differing national wealth levels and different (efficient) market outcomes. In all cases, national consumptions of the two goods are perfectly correlated because countries insure each other, to the maximum possible extent, against country-specific output shocks.

A regime of portfolio autarky, under which international asset exchanges are prohibited, need not lead to Pareto-efficient outcomes in general. In this case the home country's income is its endowment $X$, the foreign country's is $Y$, and trade is balanced in every period. If $p$ is the price of good $y$ in terms of good $x$, portfolio autarky confines the countries to the budget constraints:

$$x + py = X, \quad x^*/p + y^* = Y.$$ 

In a one-good model, a ban on asset trade would reduce both countries to complete autarky. That does not happen here because balanced trade in the two goods remains possible despite the unavailability of trade across time or across states of nature. Endowment disturbances thus continue to be transmitted abroad.

Desired consumptions under portfolio autarky are

\begin{align*}
x &= \theta X, \quad &x^* &= \theta pY, \\
y &= (1 - \theta) X/p, \quad &y^* &= (1 - \theta) Y.
\end{align*}

Market clearing requires that $x + x^* = \theta X + \theta pY = X$, so that

$$p = (1 - \theta) X/\theta Y.$$ 

This price solution and (5) yield the equilibrium consumptions:

\begin{align*}
x &= \theta X, \quad &x^* &= (1 - \theta) X, \\
y &= \theta Y, \quad &y^* &= (1 - \theta) Y.
\end{align*}

Now compare (6) with (3). The market solution under portfolio autarky is the member of the Pareto-efficient family of planning solutions corresponding to $\theta = \omega$. Thus, a planner weight of

$$\mu = 1/[1 + [(1 - \theta)/\theta] R]$$

for the home country leads to the same allocation as the market would if cross-border asset trades were prohibited.\footnote{It is easy to show that if $\theta = \frac{1}{2}$, the equivalence result holds, not only for isoelastic preferences but for any monotonic concave function of a Cobb-Douglas index.}
An example showing a Pareto-optimal market allocation under portfolio autarky has some potential implications for empirical assessments of the extent to which capital is mobile internationally. First, commodity real interest rates and other real asset returns can be identical across countries even with no capital mobility. Second, under capital mobility there may be no departures from current-account balance, even if the menu of assets traded internationally is quite limited. (In an exchange model, current-account imbalances merely substitute for diversification, and thus have no role to play if diversification is redundant.) Finally, as we argue in detail in section 5, small costs of international financial transactions could give rise to nondiversification, even in an unrestricted market setting.

2.2. Counter-examples: Nonspecialization in tradables

The foregoing theoretical result is very special, and disappears under even slight generalization. Simulations we report later examine the empirical importance of deviations from the assumptions on preferences made above. For now, however, we simply present two counter-examples that show how nonspecialization in production can lead to potential gains from international diversification. Both counter-examples modify the Lucas (1982) model's supply side.\(^8\)

As a first case, assume that the home country has a random endowment of a second good, z, along with x. The foreign country has a random endowment of the same good, z, along with y. Let Z and Z* be realizations of the home and foreign endowments of z.

Both countries' residents have the same Cobb–Douglas/isoelastic preferences with expenditure shares \(\theta_x\), \(\theta_y\), and \(\theta_z\). If \(p_z\) is the price of x in terms of z and \(p_y\) the price of y in terms of z, then it is easy to derive equilibrium prices under portfolio autarky:

\[
p_x = \frac{\theta_x(Z + Z^*)}{\theta_z X}, \quad p_y = \frac{\theta_y(Z + Z^*)}{\theta_z Y}.
\]

To check that condition (2) is generally is not satisfied, consider consumptions. Those of the x good, for example, are

\[
x = \left(\theta_z \left(\frac{Z}{Z + Z^*}\right) + \theta_x\right)X, \quad x^* = \left(\theta_z \left(\frac{Z^*}{Z + Z^*}\right) + \theta_y\right)X.
\]

\(^8\)See section 4 below and Cole and Obstfeld (1989) for examples of how demand-side modifications of subsection 2.1's model can overturn the efficiency of the portfolio-autarky equilibrium.
It is easy to see that for a given good, the international ratio of marginal utilities is constant across states of the world only if $Z$ and $Z^*$ are perfectly correlated. Without asset trade, equilibrium is therefore inefficient in general.

In the model of subsection 2.1 all output shocks are transmitted positively between countries — and in some very special cases this effect may provide perfect insurance for domestic and foreign residents alike. Now, however, shocks in common industries are transmitted negatively abroad, so that the international price mechanism need not provide automatic insurance.

The discussion brings out the important distinction between country-specific output shocks, which simultaneously affect all sectors within a country, and industry-specific shocks (for example, technological advances disseminated quickly across national borders), which affect sectors producing the same good regardless of location. If most shocks to $z$ output are industry-specific, it is plausible that $Z$ and $Z^*$ are highly correlated and that international asset trade yields little in the way of efficiency gains. But if shocks tend to be country-specific, countries gain by exchanging shares in common risky industries.\footnote{Stockman (1988) concludes from evidence on seven European countries that country-specific shocks had a substantial influence on output growth in the period after the mid-1960s. His findings thus suggest the existence of efficiency benefits from international portfolio diversification. See also Costello (1990).}

2.3. Counter-examples: Nontradable goods

It is easy to incorporate into Lucas's (1982) model stochastic home and foreign endowments, $N$ and $N^*$, of nontradables.\footnote{Stockman and Deltas (1989) extend Lucas's model in this way, focusing on exchange-rate determination. They assume, however, that nontradables and tradables affect utility separably. Our example shows the special implications of separability for optimal international portfolio diversification. As Stockman and Deltas observe, separability is implausible in any case, because tradables often come 'bundled' together with such nontradables as marketing and distribution services.} Preferences are still Cobb–Douglas/isoelastic, with expenditure shares $\theta_x$, $\theta_y$, and $\theta_n$, and a risk-aversion coefficient $R$ common to both countries.

In a balanced-trade equilibrium the two national markets for nontradables clear while demands for tradables are:

$$x = \left[ \frac{\theta_x}{1 - \theta_n} \right] X, \quad x^* = \left[ \frac{\theta_y}{1 - \theta_n} \right] X,$$

$$y = \left[ \frac{\theta_x}{1 - \theta_n} \right] Y, \quad y^* = \left[ \frac{\theta_y}{1 - \theta_n} \right] Y.$$

The expressions above are similar to those in (6) and likewise imply that consumptions of each tradable are perfectly correlated internationally. Of
course, national consumptions of nontradables are N and N*, and thus have an arbitrary correlation.

If the planner optimizes over tradable consumptions only, condition (2) still characterizes optimal allocations, except that marginal utilities of tradables may now depend on N and N*. This dependence generally will upset the efficiency of the incomplete-markets allocation. To see this, note that the marginal-utility ratio in (2) would be \( \Omega(N_t/N_t^*)^{\theta_x(1-R)} \), where \( \Omega \) is a constant that depends on \( \theta_x \) and \( \theta_Y \). But unless N and N* are perfectly correlated, or unless the utility function is separable in consumptions \( \Omega = 1 \), the expression above is not constant across states of nature. Nontradable goods may therefore imply additional efficiency benefits from international risk sharing.\(^\text{11}\)

3. A logarithmic investment model

Investment is now introduced to add an intertemporal dimension to the inquiry. The basic setup comes from the closed-economy analysis of Long and Plosser (1983) (which is easily reinterpreted as an open-economy analysis in the complete-markets case). As before, the model is worked out twice, once under free international trade in a complete set of contingent claims, once under the assumption of portfolio autarky. Shocks to production technologies are the underlying source of uncertainty.

The conditions under which portfolio autarky is efficient appear even more stringent than in the models of section 2. In this section we need to assume that preferences are logarithmic as well as intertemporally separable. The assumption of a unit elasticity of intertemporal substitution plays a role below that corresponds to section 2's assumption of a unit elasticity of intratemporal substitution.\(^\text{12}\)

3.1. Setup of the model: Complete markets

The home and foreign countries are now specialized in their production of the two goods, rather than in endowments. Output of a good in period \( t + 1 \), say, depends on a random period-(\( t + 1 \)) productivity disturbance and period-\( t \) inputs of both goods. Let \( k_{XX_t} \) \( (k_{XY_t}) \) and \( k_{XY_t} \) \( (k_{YY_t}) \) be the home (foreign)\(^\text{11}\) Nontradables also may influence the statistical correlation between broad national consumption measures. The models examined earlier imply that national consumption levels are more highly correlated than they are in reality. But when a significant portion of each country's consumption falls on nontradables, aggregate national consumptions need not be highly correlated, financial integration notwithstanding (Stockman and Dellas (1989)).

\(^\text{12}\) In Cole and Obstfeld (1989), we study two additional investment models in which there are no gains from international risk sharing when preferences are Cobb–Douglas/isoelastic. In the first, production functions are linear in capital and production shocks are i.i.d.; in the second, strong symmetry conditions are imposed on preferences and production functions.
inputs of goods $x$ and $y$ into the production of period-$(t + 1)$ outputs. As in Long and Plosser (1983), this investment depreciates completely in the production process.

Home and foreign production functions are

$$
\ln X_{t+1} = \ln \xi_{t+1}^X + \gamma_{XX} \ln k_{XXt} + \gamma_{XY} \ln k_{XYt},
$$

$$
\ln Y_{t+1} = \ln \xi_{t+1}^Y + \gamma_{YX} \ln k_{YXt} + \gamma_{YY} \ln k_{YYt}.
$$

Above, $\gamma_{XY} = 1 - \gamma_{XX}$ and $\gamma_{YX} = 1 - \gamma_{YY}$ (a constant-returns assumption). The multiplicative productivity disturbance $\zeta = (\xi^X, \xi^Y)$ is Markovian with positive support.

Let $x_t$ ($x_t^*$) and $y_t$ ($y_t^*$) again stand for home (foreign) consumptions of the two goods. Home residents maximize

$$
U_0 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \theta \ln x_t + (1 - \theta) \ln y_t \right] \right\}
$$

and foreign residents maximize

$$
U_0^* = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \theta^* \ln x_t^* + (1 - \theta^*) \ln y_t^* \right] \right\}.
$$

Notice that consumption preferences may differ internationally.\(^{13}\)

Pareto-optimal allocations solve the problem of maximizing a social welfare function of the form $\mu \cdot U_t + (1 - \mu) \cdot U_t^*$, subject to resource constraints. As before, these allocations correspond to complete-markets equilibria conditional on different initial international distributions of wealth. The planning problem can be solved in two stages. First, allocate given world aggregate consumption levels, $C_{XX}$ and $C_{YY}$, between residents of the two countries. (This stage corresponds to the market process of sharing consumption risks optimally given the global distribution of wealth.) Second, choose the aggregate consumption levels $C_{XX}$ and $C_{YY}$ optimally at each point in time.

Consider stage 1 first, the same static problem examined in section 2. In this stage the planner maximizes the weighted sum

$$
\mu \cdot [\theta \ln x_t + (1 - \theta) \ln y_t] + (1 - \mu) \cdot [\theta^* \ln x_t^* + (1 - \theta^*) \ln y_t^*],
$$

(8)

\(^{13}\)Cantor and Mark (1988), in an interesting paper, also study a logarithmic two-country investment model, but they assume that the countries' outputs are perfect substitutes. Their model, which posits an internationally immobile labor force, is solvable in closed form only when production shocks are i.i.d. (The solution approach used in this section becomes inapplicable.)
subject to the constraints
\[ x_t + x_t^* \leq C_{X_t}, \quad y_t + y_t^* \leq C_{Y_t}. \]

The resulting allocation rules are:
\[
\begin{align*}
x_t &= \mu \theta C_{X_t} / \phi, \\
x_t^* &= (1 - \mu) \theta^* C_{X_t} / \phi, \\
y_t &= \mu (1 - \theta) C_{Y_t} / (1 - \phi), \\
y_t^* &= (1 - \mu) (1 - \theta^*) C_{Y_t} / (1 - \phi),
\end{align*}
\]

where
\[
\phi = \mu \theta + (1 - \mu) \theta^*, \quad 1 - \phi = \mu (1 - \theta) + (1 - \mu) (1 - \theta^*). \tag{10}
\]

The preceding solution defines an indirect planner utility function that depends on the world aggregates \( C_{X_t} \) and \( C_{Y_t} \). Direct substitution of (9) and (10) into (8) shows that (aside from an additive constant) this indirect utility function equals
\[
W(C_{X_t}, C_{Y_t}) = \phi \ln C_{X_t} + (1 - \phi) \ln C_{Y_t}. \tag{11}
\]

Eq. (11) brings one back to stage 2 of the planning problem, choosing the optimal aggregate consumptions of the goods. This stage-2 problem takes the form:
\[
\text{maximize } E_0 \left\{ \sum_{t=0}^{\infty} \beta^t W(C_{X_t}, C_{Y_t}) \right\}, \tag{12}
\]

subject to the constraints
\[
C_{X_t} + k_{XX_t} + k_{YX_t} \leq X_t, \quad C_{Y_t} + k_{XY_t} + k_{YY_t} \leq Y_t,
\]

and given the intertemporal tradeoffs defined by the national investment functions.

When posed as in (12), the aggregate planning problem is exactly the problem studied by Long and Plosser (1983). The solution to stage 2 therefore can be lifted from their paper with no modification. That solution is
\[
\begin{align*}
C_{X_t} &= (\phi / \kappa_X) X_t, \\
C_{Y_t} &= [(1 - \phi) / \kappa_Y] Y_t, \\
k_{jXX_t} &= (\beta \kappa_j \gamma_{jX} / \kappa_X) X_t, \\
k_{jXY_t} &= (\beta \kappa_j \gamma_{jY} / \kappa_Y) Y_t, \\
\end{align*}
\]

where
\[
\begin{align*}
\kappa_X &= \frac{[\phi (1 - \beta \gamma_{YY}) + (1 - \phi) \beta \gamma_{XX}]}{\Delta}, \\
\kappa_Y &= \frac{[(1 - \phi) (1 - \beta \gamma_{XX}) + \phi \beta \gamma_{XY}]}{\Delta},
\end{align*}
\]

\[
\Delta = (\phi \beta \gamma_{XX} + (1 - \phi) \beta \gamma_{YY}).
\]
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The parameters $\kappa_X$ and $\kappa_Y$ are the coefficients of $\ln X_t$ and $\ln Y_t$ in the planner's period-$t$ value function.

Allocation rules (9) and (13) have several important implications about world equilibrium with complete markets (given the classes of utility and production functions under study here). Specifically, consumption of every good is perfectly correlated across countries, as in Lucas's endowment model. Thus, each country's consumption of a good is perfectly correlated with world consumption of that good, which is proportional in turn to world output. Investment of every good is also perfectly correlated across countries, and proportional to current output.

In comparing the foregoing equilibrium with the one that results when there is no asset trade, it is useful to derive a key shadow price associated with the optimal allocation, the shadow price of good $y$ in terms of good $x$, $p_t$. At any time $t$, $p_t$ is the marginal rate of substitution in consumption of good $x$ for good $y$,

$$p_t = (1 - \phi)C_{Xt}/\phi C_{Yt} = \kappa_Y X_t/\kappa_X Y_t.$$  

This price depends on $\mu$ only if $\theta \neq \theta^*$ [see (10) and (14)].

3.2. Equilibrium under portfolio autarky

Market equilibrium is found in two steps. First, solve the maximization problems of home and foreign social planners who take the stochastic process generating the terms of trade, $(p_t)$, as exogenously given. This step yields price-dependent consumption and investment demands for the two goods. Second, compute the terms of trade that clear world goods markets. This step yields reduced-form consumption and investment demands that can be compared with (9) and (13).

Consider first the problem of a home social planner. Our conjecture is that this planner's value function takes the form

$$V(X_t, \xi_t^X, p_t) = \kappa \ln X_t + J(\xi_t^X) + H(p_t)$$

(up to an additive constant), where $p_t$, the price of good $y$ in terms of good $x$, is given. Bellman's principle states that $V(X_t, \xi_t^X, p_t)$ solves the problem of

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Dellas (1986) studies a model similar to this one and draws on the Long-Plosser solution to describe its equilibrium, even though his model assumes balanced international trade. We have been unable to find a direct justification for this solution procedure.
maximizing
\[
\theta \ln x_t + (1 - \theta) \ln y_t + \beta E_t \{ V(X_{t+1}, x_t^{X_{t+1}}, p_{t+1}) \},
\]
subject to the \( X_{t+1} \) production function and the budget constraint
\[
x_t + k_{XXt} + p_t( y_t + k_{XYt}) = X_t.
\]
Given conjecture (16), first-order conditions for the problem are
\[
\begin{align*}
\frac{\theta}{x_t} &= \lambda_t, \\
\frac{1 - \theta}{y_t} &= \lambda_t p_t, \\
(1 - \theta)/Y_t &= \lambda_t p_t, \\
\beta k_X \gamma_X / k_{XXt} &= \lambda_t, \\
\beta k_Y \gamma_Y / k_{XYt} &= \lambda_t p_t,
\end{align*}
\]
where \( \lambda_t \) is the Lagrange multiplier on the balanced-trade budget constraint. From the envelope condition, \( \partial V(X_t, x_t^{X_t}, p_t)/\partial X_t = \kappa/X_t = \lambda_t \), we derive
\[
\kappa = \lambda_t X_t.
\]
Combination of (17) and (18) with the economy's budget constraint leads to the solution \( \kappa = 1/(1 - \beta) \) and to the following demand functions:
\[
\begin{align*}
x_t &= \theta(1 - \beta) X_t, \\
y_t &= (1 - \theta)(1 - \beta) X_t/p_t, \\
k_{XXt} &= \beta \gamma_X X_t, \\
k_{XYt} &= \beta \gamma_Y X_t/p_t.
\end{align*}
\]
The corresponding demand functions for the foreign country are the analogues of those listed in (19):
\[
\begin{align*}
x_t^* &= \theta^*(1 - \beta) p_t Y_t, \\
y_t^* &= (1 - \theta^*)(1 - \beta) Y_t, \\
k_{XYt} &= \beta \gamma_X P_t Y_t, \\
k_{YYt} &= \beta \gamma_Y Y_t.
\end{align*}
\]
Equilibrium in the market for the \( x \) good determines \( p_t \):
\[
x_t + x_t^* + k_{XXt} + k_{XYt} = X_t.
\]
Eqs. (19) and (20) imply that the condition above holds when
\[
p_t = \frac{X_t}{Y_t} \times \frac{(1 - \theta)(1 - \beta) + \beta \gamma_X}{\theta^*(1 - \beta) + \beta \gamma_Y}. \quad (21)
\]
\section{3.3. A redundancy proposition}
When combined with the price function (21), (19) and (20) show that the portfolio-autarky case bears many empirical similarities to the complete-
markets case. In fact, the equilibrium with restricted asset trade implies the same resource allocation as a particular optimal plan.

We show this equivalence by constructing an optimal plan that calls for the incomplete-markets allocation. Eq. (15), which describes the set of all shadow prices \( p_t \) generated by efficient allocations, can be written

\[
p_t = \frac{X_t}{Y_t} \times \frac{(1 - \phi)(1 - \beta) + \beta \gamma_{XY}}{\phi(1 - \beta) + \beta \gamma_{YY}}.
\]

(22)

Compare (21) and (22). They are the same for a planner weight of

\[
\mu = \frac{\theta^*(1 - \beta) + \beta \gamma_{YY}}{(1 - \theta + \theta^*)(1 - \beta) + \beta (\gamma_{YY} + \gamma_{XY})}.
\]

(23)

In the special case \( \phi = \theta = \theta^* \), it is easy to verify directly that the planner weights producing the autarkic market allocation are \( \mu = \kappa_X(1 - \beta) \) and \( 1 - \mu = \kappa_Y(1 - \beta) \), where \( \kappa_X \) and \( \kappa_Y \) are given by (14).

If the relative price of the two goods under portfolio autarky equals the price generated by the plan, however, eqs. (15), (19), (20), and (23) can be used to show directly that both sets of arrangements lead to the same allocation. Efficiency is therefore assured even without asset trade.

4. How big are the gains from risk sharing? Some numerical results

We have examined some very special models in which international portfolio diversification yields no welfare benefits at all. We next ask whether empirical departures from the specific assumptions in our examples imply significant gains from international risk sharing. As a first step, we report numerical simulations partially calibrated to United States and Japanese data.

The results are limited to the case of pure exchange, and their implications about reality therefore need to be interpreted with caution. Strictly speaking, our calculations apply only to a hypothetical pure endowment economy whose exogenous output process is calibrated using actual data. The results are primarily useful as an indication of likely orders of magnitude.

4.1. Preferences and equilibrium

Even within a pure exchange setting, several possible ways of relaxing the assumptions above would upset asset-trade redundancy. Below, we continue to assume that both countries' endowments are specialized, but we relax the Cobb–Douglas preference assumption under which terms of trade are unit-
elastisch with respect to relative output. One limiting case that we do cover allows the countries' outputs to be perfect substitutes, so that terms-of-trade effects are absent.

Specifically, we now assume a constant elasticity of substitution (CES)/isoelastic utility specification:

\[ u(x, y) = \left[ (x^\rho + y^\rho)^{1/\rho} \right]^{1-R} / (1 - R), \quad \rho \leq 1. \]

Goods \( x \) and \( y \) are perfect substitutes in consumption if \( \rho = 1 \); \( \rho = 0 \) is the Cobb–Douglas case, with equal commodity weights of \( \frac{1}{2} \).

Absent asset trade, desired consumption levels are

\[
\begin{align*}
    x &= X / \left[ 1 + p^{\rho/(\rho - 1)} \right], \\
    y &= p^{1/(\rho - 1)} X / \left[ 1 + p^{\rho/(\rho - 1)} \right], \\
    x^* &= pY / \left[ 1 + p^{\rho/(\rho - 1)} \right], \\
    y^* &= p^{\rho/(\rho - 1)} Y / \left[ 1 + p^{\rho/(\rho - 1)} \right].
\end{align*}
\]

Market clearing yields the equilibrium price function

\[ p = (X/Y)^{1-\rho}. \]

The case \( \rho < 0 \) implies 'immiserizing growth' and probably is unrealistic. Our simulations therefore assume \( \rho \in [0, 1] \).

4.2. Numerical methodology

Let \( \{\xi_t^X\} \) and \( \{\xi_t^Y\} \) be exogenous stochastic processes. National outputs are assumed to follow

\[
\begin{align*}
    X_{t+1} &= (1 + \xi_t^X)X_t, \\
    Y_{t+1} &= (1 + \xi_t^Y)Y_t.
\end{align*}
\]

Each process in (26) is a two-state Markov process, with the two states corresponding to 'high growth' and 'low growth'. The state of the world economy as a whole on date \( t \) is given by the vector \( (\xi_t^X, \xi_t^Y) \), which can take four possible values.\(^{15}\)

We calibrate our simulation model so that the mean, standard deviation, and first lagged autocorrelation of either country's output growth rate equal those of the United States annual per capita output growth rate over the years 1968–1987. The numbers are 1.8 percent per year (mean growth), 2.7 percent per year (standard deviation of growth), and 0.102 (first lagged autocorrelation of growth). It is also assumed that the contemporaneous

\(^{15}\) The simulation model is a two-country version of the model used by Mehra and Prescott (1985).
correlation coefficient between the two countries' growth rates equals that between U.S. and Japanese per capita growth over the same two decades (that is, 0.375).\textsuperscript{16}

Calibration amounts to choosing the two possible realizations of the Markov growth processes and the probability entries in the state transition matrix. The four possible states for the world economy are taken to be:

\begin{align*}
\text{State 1:} & \quad \varepsilon^X = 0.045, \quad \varepsilon^Y = 0.045, \\
\text{State 2:} & \quad \varepsilon^X = 0.045, \quad \varepsilon^Y = -0.009, \\
\text{State 3:} & \quad \varepsilon^X = -0.009, \quad \varepsilon^Y = 0.045, \\
\text{State 4:} & \quad \varepsilon^X = -0.009, \quad \varepsilon^Y = -0.009.
\end{align*}

Let $\pi_{ij}$ denote the probability of moving to state $j$ next period when the current state is $i$. The transition matrix $\Pi = [\pi_{ij}]$ is

\[
\Pi = \begin{bmatrix}
0.600 & 0.050 & 0.050 & 0.300 \\
0.110 & 0.222 & 0.558 & 0.110 \\
0.110 & 0.558 & 0.222 & 0.110 \\
0.300 & 0.050 & 0.050 & 0.600
\end{bmatrix},
\]

with an implied steady-state distribution described by the unconditional probabilities $[\pi_1 \pi_2 \pi_3 \pi_4] = [0.344 \ 0.156 \ 0.156 \ 0.344]$.

The model has been set up so that the two countries are perfectly symmetric under portfolio autarky. In particular, the transition matrix $\Pi$ implies a symmetric joint distribution for the countries' growth rates. The symmetry assumption is deliberate: it implies that the opening of asset trade would move the world economy to a perfectly pooled equilibrium in which countries hold equal wealth. In the numerical results, it is with this perfectly pooled equilibrium that the autarkic equilibrium is compared.

\section*{4.3. Results}

We assume that the two countries always start out with predetermined base output levels $\bar{X}$ and $\bar{Y}$, where $\bar{X} = \bar{Y}$ is imposed to maintain symmetry. A single realized history for the world economy is generated as follows. In period $t = 0$, an initial pair of growth rates, $(\varepsilon^X_0, \varepsilon^Y_0)$, is drawn from the steady-state distribution. Consumptions for $t = 0$ are then determined according to the assumptions about financial integration, given the output levels $(1 + \varepsilon^X_0)\bar{X}$ and $(1 + \varepsilon^Y_0)\bar{Y}$. Subsequent growth rates are draws from the conditional distribution defined by the transition matrix $\Pi$, and these generate subsequent output and consumption levels.

\textsuperscript{16}Data comes from the Penn World Table (Mark 5). Use of output rather than consumption moments only magnifies our welfare-loss estimates.
Table 1
Welfare loss due to a ban on international diversification (fraction of national product per year).

<table>
<thead>
<tr>
<th>R</th>
<th>0.25</th>
<th>0.50</th>
<th>0.75</th>
<th>1.00</th>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
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<td>0.000347</td>
<td>0.000906</td>
<td>0.001722</td>
</tr>
<tr>
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<td>(0.000174)</td>
<td>(0.000261)</td>
<td>(0.000349)</td>
</tr>
<tr>
<td>4</td>
<td>0.000120</td>
<td>0.000619</td>
<td>0.001497</td>
<td>0.002760</td>
</tr>
<tr>
<td></td>
<td>(0.000074)</td>
<td>(0.000148)</td>
<td>(0.000223)</td>
<td>(0.000300)</td>
</tr>
<tr>
<td>6</td>
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<td>0.000752</td>
<td>0.001790</td>
<td>0.003284</td>
</tr>
<tr>
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<td>(0.000132)</td>
<td>(0.000201)</td>
<td>(0.000272)</td>
</tr>
<tr>
<td>8</td>
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<td>0.000829</td>
<td>0.001965</td>
<td>0.003605</td>
</tr>
<tr>
<td></td>
<td>(0.000060)</td>
<td>(0.000122)</td>
<td>(0.000186)</td>
<td>(0.000255)</td>
</tr>
<tr>
<td>10</td>
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<td>0.000882</td>
<td>0.002088</td>
<td>0.003841</td>
</tr>
<tr>
<td></td>
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<td>(0.000116)</td>
<td>(0.000177)</td>
<td>(0.000245)</td>
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<tr>
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<td>0.002469</td>
<td>0.004851</td>
</tr>
<tr>
<td></td>
<td>(0.000057)</td>
<td>(0.000120)</td>
<td>(0.000199)</td>
<td>(0.000310)</td>
</tr>
</tbody>
</table>

*For a given CES utility function parameter \( \rho \) and risk aversion coefficient \( R \), the reported number is the fraction by which base-year output must be reduced to yield a welfare loss equal to that caused by a ban on international asset trade. Expected utility levels are calculated as the average of utility realizations in 10,000 independent replications of a symmetric two-country world economy in which national output growth rates follow a two-state Markov process. (Approximate standard errors appear in parentheses below the output-loss estimates.)

Estimates of home-country expected utilities under portfolio autarky and under perfect pooling, denoted \( \hat{U}^A(\bar{X}) \) and \( \hat{U}^P(\bar{X}) \), are averages of the lifetime utility levels realized in 10,000 independent histories. (Regardless of market structure, expected utility at home equals expected utility abroad.) The calculations assume that \( \beta = 0.98 \) and that the economy's horizon is 50 periods.

Our measure of welfare cost is the fraction \( \delta \) by which the base outputs \( \bar{X} \) and \( \bar{Y} \) would have to be reduced in the perfectly pooled case to leave people with the expected utility attainable when portfolio autarky is imposed. The fraction \( \delta \) is estimated as the solution to \( \hat{U}^A(\bar{X}) = \hat{U}^P[(1 - \delta)\bar{X}] \), i.e., as the permanent percentage reduction in average global product equivalent to the prohibition of international portfolio diversification.

Table 1 reports the estimates \( \delta \) for CES coefficients ranging from 0.25 to 1.0 and for risk-aversion coefficients ranging from 2 to 30. (Approximate standard errors appear in parentheses.) The most striking fact revealed by the table is how small the gains from international asset trade are in this pure

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17We estimated expected utility under both regimes, rather than calculating it with closed-form solutions such as Mehra's (1988), because those solutions do not readily apply to the autarky case.
exchange model. The largest welfare loss reported, 0.49 percent of output per year, occurs when \( \rho = 1 \) and \( R = 30 \). Recall that when \( \rho = 1 \), national outputs are perfect substitutes and endogenous terms-of-trade fluctuations therefore provide no insurance against endowment risk. Estimates based on \( \rho = 1 \) might be relevant for a small country producing an output that is also produced by many foreign producers; but welfare losses are likely to be smaller for larger economies. In any case, even a yearly welfare loss equivalent to 0.49 percent of output is not crushingly large.\(^{18}\)

Values of \( \rho \) between 0.25 and 0.75 seem relevant for most industrial countries. The size of \( R \) found in the empirical literature is generally 4 or below [e.g., Mehra and Prescott (1985), Pindyck (1988)]. For plausible parameters, table 1 thus shows a yearly welfare loss likely to be around 0.20 percent of output.

5. Conclusions, qualifications, and future research

If our low estimates of the welfare gains from international risk pooling are accurate, small impediments to asset trade could discourage a large volume of two-way capital flows between industrial countries, and might also limit cross-border diversification. Since current-account movements can in some respects substitute for international diversification, small gains from diversification could help explain the generally small scale of current-account imbalances as well. Strong conclusions cannot be drawn from simulations as rudimentary as ours. A conservative inference, however, is that limited gains from asset trade offer a potential clue to the puzzle surrounding the measurement of capital mobility among industrial economies.

More detailed empirical work is needed before a solution to the puzzle involving small trade gains and small transaction costs can be accepted with any confidence. In particular, future research will have to extend simple simulation examples like ours along several dimensions:

(1) Investment and intertemporal trade. The key limitation of our simulation analysis is that it neglects investment. Although there exist investment models in which international asset trade does not enlarge the set of consumption opportunities, these models are very special and capture only part of the role investment plays in reality. Trade across time could provide

\(^{18}\)See Lucas (1987) for discussion of other pertinent welfare comparisons. Future work should decouple risk aversion and intertemporal substitutability, as in the preferences studied by Epstein and Zin (1989) and Weil (1990). For the output processes and preference parameter values assumed here, higher \( R \), while worsening the welfare loss through its risk-aversion role, actually mitigates that loss through its intertemporal-substitution role. As a rough indication, the loss for \( R = 10, \rho = 1 \), would be around 2.25 times its reported size were the intertemporal substitution parameter held constant at 2.
significant welfare benefits even when pure trade across states of nature does not, and could conceivably amplify the gains from diversification. A complete evaluation of asset-trade gains among industrial countries requires a realistic account of investment.

Two recent simulation studies do follow up on our idea that asset-trade gains among the richer countries are limited. Mendoza (1990) studies the costs of capital controls in a small-country, one-consumption-good model with investment. Calibration to the Canadian economy produces welfare-loss estimates of the same order of magnitude as those reported above. Backus, Kehoe, and Kydland (1991) calibrate a two-country, one-good simulation model with investment to data from the U.S. and an aggregate of six European countries. They find costs of autarky similar to those we report in section 4, as well as large effects of small trading costs.  

(2) Uncertainty and transaction costs. Realistic modeling of transaction costs within a general-equilibrium framework is likely to generate hysteresis in portfolios and in physical investments. Constantinides's (1986) work on individual portfolio choice with proportional transaction costs suggests that small costs can create wide 'bands of inaction' while inflicting small utility losses and causing small departures from equilibrium asset pricing. These findings seem capable of explaining why global portfolio diversification has been a slow process at the same time that international interest-rate linkages have been tight. Similar models could explain limited direct investment flows (Dixit 1991).

(3) Additional shocks. Our models focus exclusively on productivity shocks; yet the presence of additional shocks – fiscal, monetary, or preference shocks – could increase the scope for trade gains. To the extent that such omitted factors affect welfare through their consumption effects, however, their explicit inclusion seems unlikely to add much.  

(4) Private versus aggregate social gains. It was argued above that the aggregate social gains from risk sharing among industrialized countries may be small. But the gains perceived by individuals and those accruing to society may diverge for several reasons. For example, even when the social gains from diversification are small, tax avoidance may result in a large volume of two-way (sometimes unreported) international financial flows. Furthermore,

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19Since these models are designed to fit certain moments of actual data, their results might best be interpreted as measuring the cost of moving from the current world asset-market structure – which contains some imperfections – to autarky. Brennan and Solnik (1989), using data on current accounts, calculate large gains from intertemporal trade among a group of industrial countries. Obstfeld (1990) argues that these results overstate the gains by as much as two orders of magnitude.

20Stockman and Svensson (1987) describe a model with a richer menu of disturbances.
if domestic capital markets are imperfect, aggregate output variability may greatly understate individual income variability; and some individuals or income groups in the economy may gain disproportionately from international diversification. Such gains would also work against our hypothesis. Further research that relaxes our representative national agent assumption clearly is warranted.21

(5) Endogenous growth effects. International financial integration might affect output growth rates, as suggested by the recent literature on endogenous long-run growth.22 Opening national capital markets to foreign competition might improve the efficiency of domestic financial intermediation, with permanent effects on investment and growth. Or, direct investment by foreigners could increase the speed with which technological innovations are disseminated between countries. The means through which financial integration promotes growth might well involve externalities not captured by private investors.23 Our hunch is that growth effects such as these are likely to be a quantitatively important benefit of global financial integration.

(6) Developing countries. The paper’s analysis applies most readily to industrialized countries, and has not addressed the possibility of substantial intertemporal trade gains between countries at very different stages of economic development. However, more elaborate models incorporating investment can help account for the gains reaped in the past from foreign capital flows to developing countries. We should also note that even though the social gains from portfolio diversification among developed economies may be small, this is not the case for most developing countries. For example, Brazil’s consumption (measured in real per capita terms) is much less correlated with world consumption than are the consumptions of most industrial countries. Developing countries would gain substantially from greater integration into world financial markets.

Further research into all of these questions is needed to resolve the riddle in the data and to evaluate the performance of global financial markets more generally.

21 Thus, Golub (1990b) finds that the aggregate social gain from risk sharing between Japan and the U.S. is small, but that some private risk, especially the risk connected with corporate-profit income, could be reduced substantially by international pooling. A related finding concerns the substantial measured gains from diversifying into foreign equities [e.g., French and Poterba (1990)]. A key unresolved issue is the extent to which the measured private risk reductions can be obtained through domestic asset markets.

22 See Romer (1990) for a survey.

23 If this is so, our suggested interpretation of the capital-mobility puzzle could still stand: while growth effects may alter our assessment of the gains from asset trade, they can influence private capital flows only to the extent that they are reflected in the private returns that investors perceive.
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