Sources of Lifetime Inequality

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Abstract

Is lifetime inequality mainly due to differences across people established early in life or to differences in luck experienced over the lifetime? We answer this question within a model with risky human capital. We find that as of age 20 differences in *initial conditions* account for more of the variation in lifetime utility and in lifetime wealth than do differences in shocks received over the lifetime. Among initial conditions, variation in initial human capital is relatively more important than variation in learning ability for determining how an agent fares in life. However, differences in learning ability are key in accounting for the increase in earnings dispersion for a cohort over the lifetime observed in US data.

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1 Introduction

To what degree is lifetime inequality due to differences across people established early in life as opposed to differences in luck experienced over the lifetime? Among the initial conditions, which ones are the most important? A convincing answer to these questions is of fundamental importance. First, and most simply, an answer serves to contrast the importance of the myriad policies directed at modifying initial conditions or providing insurance for initial conditions against those directed at shocks over the lifetime. Second, a discussion of lifetime inequality cannot go too far before discussing which type of initial condition is the most critical for determining how one fares in life. Third, a useful framework for answering these questions should also be central in the analysis of a wide range of policies considered in fields as disparate as macroeconomics, labor economics and public economics. In these fields policies are often analyzed by abstracting from an important role for luck over the lifetime.

This paper quantifies the importance of different proximate sources of lifetime inequality. The analysis focuses on the inequality in lifetime utility and lifetime wealth. The former measure is theoretically well motivated since in standard economic theory agents maximize expected lifetime utility. The focus on lifetime wealth is important since in standard life-cycle theory an agent's lifetime budget constraint is determined by lifetime wealth.

We start from the premise that a useful framework for analyzing lifetime inequality needs to be consistent with some critical distributional facts over the life cycle. To this end, we document for US males how mean earnings and how measures of earnings dispersion evolve for a typical cohort as the cohort ages. We find that mean earnings are hump shaped over the working lifetime and that earnings dispersion increases over most of the working lifetime. These features of US data have been widely documented in other studies and have been viewed as key facts that a theory of inequality should explain.[1] In

[1]Mincer (1974) documented related patterns in US cross-section data. Many studies use repeated cross-section data at the individual or the household level to document these patterns for a cohort as the cohort ages (e.g. Deaton and Paxson (1994), Storesletten, Telmer and Yaron (2004), Heathcote, Storesletten and Violante (2005a) and Huggett, Ventura and Yaron (2006)).

particular, the rise in earnings dispersion has been viewed as reflecting an important role for idiosyncratic shocks as well as a role for initial differences in learning ability.

We view lifetime inequality through the lens of a risky human capital model. Within the model, individuals differ in terms of three *initial conditions:* initial human capital, learning ability and financial wealth. As individuals age, they accumulate human capital by optimally dividing their available time between market work and human capital accumulation. Human capital and labor earnings are *risky* as human capital is subject to uninsurable, idiosyncratic shocks each period.

Our model produces the hump-shaped mean earnings profile by a standard human capital argument. Early in life earnings are low as agents allocate time to accumulating human capital. Earnings rise as human capital accumulates and as a greater fraction of time is devoted to market work. Earnings fall later in life as human capital depreciates and little or no time is put into producing new human capital.

Two forces account for the increase in earnings dispersion. One force is that agents differ in learning ability. Agents with higher learning ability have steeper sloped mean earnings profiles than low ability agents, other things equal.[2] The other force is that agents differ in idiosyncratic, human capital shocks received over the life cycle. To identify the contribution of each force, we exploit the fact that the model implies that late in life little or no new human capital is produced. As a result, moments of the change in wage rates for these agents are entirely determined by shocks, independently of all other model parameters. Thus, we estimate the shock process from US data using these moments. Given an estimate of the shock process, we choose the initial distribution of financial wealth, human capital and learning ability across agents to best match the earnings facts described above, given all other model parameters.[3] We then use these estimates of initial conditions and shocks in our analysis of the proximate sources of lifetime inequality.

[2]This mechanism is supported by the literature, reviewed by Card (1999), on the shape of the mean age-earnings profiles by years of education. It is also supported by the work of Baker (1997) and Guvenen (2005). They estimate a statistical model of earnings and find that permanent differences in individual earnings growth rates are negatively correlated with initial earnings levels and produce large differences in the present value of earnings. Our model endogenously produces these relationships.

[3]Since a measure of financial wealth is observable, we choose the tri-variate initial distribution to match the unconditional distribution of financial wealth among young males.

We find that the majority of the variation in lifetime utility and in lifetime wealth as of a real-life age of 20 is due to differences in initial conditions. Among initial conditions, we find that differences in initial human capital are the most important source of variation in expected lifetime utility or in expected lifetime wealth. Differences in initial human capital serve to raise or lower the lifetime expected earnings profile. Differences in learning ability rotate the expected earnings profile. Higher learning ability lowers initial earnings but raises future expected earnings, other things equal. We find that differences in learning ability produce much of the rise in earnings dispersion over the lifetime within the model, given our estimate of the magnitude of human capital risk. Eliminating learning ability differences leads, other things equal, to an almost flat earnings dispersion profile. The resulting profile is not flat because shocks are small. Instead, it occurs because initial human capital differences always lead to falling earnings dispersion for a cohort over time, absent learning differences and absent shocks.

A leading and alternative view of lifetime inequality to the one analyzed in this paper is presented in Storesletten et. al. (2004). Their model is an incomplete-markets model in which labor earnings is exogenous.[4] They conclude that within their model, highly persistent earnings shocks arriving each period are needed to produce the rise in consumption dispersion with age documented by Deaton and Paxson (1994). In fact, the degree of persistence in shocks to earnings that is required to reproduce the consumption dispersion observations within their model is approximately the persistence that they estimate from US data. In their model about half of the variation in lifetime earnings or in lifetime utility is due to differences in shocks received over the lifetime.[5]

We note three difficulties related to this view. First, the rise in US consumption dispersion with age is much larger using data from 1980-90 (the period examined in

[4]Models with these properties have been used in the macroeconomic literature on economic inequality. Some of this literature is surveyed in Quadrini and Rios-Rull (1997). Recent papers in the literature include Castañeda, Diaz-Jimenez and Rios-Rull (2003), Conesa and Krueger (1999), DeNardi (2004), French (2004), Heathcote, Storesletten and Violante (2005b), Imrohoroglu, Imrohoroglu and Joines (1995), Huggett (1996), Huggett and Ventura (1999), Krueger and Perri (2005), Quadrini (2000) and Storesletten et. al. (2004), among many others.

[5]In the context of a career-choice model, Keane and Wolpin (1997) find a more important role for initial conditions. They find that unobserved heterogeneity realized at age 16 accounts for about 90 percent of the variance in lifetime utility.

Deaton and Paxson (1994)) than it is for data covering longer and more recent time periods - see Heathcote et. al. (2005a) and Slesnick and Ulker (2005). Thus, their model generates too much of a rise in consumption dispersion with age to be consistent with recent evidence. Second, their model may over estimate the importance of idiosyncratic earnings risk after entering the labor market because all the rise in earnings dispersion with age is attributed to shocks. Baker (1997) and Guvenen (2005) provide evidence for economically important differences in permanent earnings growth rates across people. These permanent growth differences emerge naturally in the human capital framework we explore. Third, the model is not useful for some purposes. Specifically, since earnings are exogenous, the model gives up on theorizing about the underlying sources of earnings inequality. Thus, the model has nothing to say about how policy may affect inequality in lifetime labor earnings or may affect welfare through earnings. Models with exogenous wages (e.g. Heathcote et. al. (2005b)) face this criticism, but to a lesser extent, since most earnings variation is attributed to wage variation. In our view, it is worthwhile to pursue a more fundamental approach that in essence endogenizes these wage differences via human capital theory.

The paper is organized as follows. Section 2 presents the model. Section 3 documents US earnings distribution facts and infers properties of shocks from wage rate data. Section 4 sets model parameters. Section 5 and 6 presents results for earnings distribution dynamics produced by the model and for the analysis of sources of lifetime inequality.

2 The Model

We study the decision problem of an agent in an otherwise standard incomplete-markets framework, but modified so that earnings are endogenously determined by human capital decisions.[6] An agent's preferences over consumption allocations are determined by a calculation of expected utility as indicated below. Consumption $c_j(z^j)$ in period j is risky

[6]The model generalizes Ben-Porath (1967) to allow for risky human capital. Risky human capital is modeled by extending the two-period models of Levhari and Weiss (1974) and Eaton and Rosen (1980) to a multi-period setting. Krebs (2004) also analyzes a multi-period model of human capital with idiosyncratic risk. Our work differs by its focus on lifetime inequality, and by how the model is related to data, among other differences.

as it depends on the j-period history of human capital shocks z^j. The set of possible j-period histories is denoted $Z^j \equiv \{z^j = (z_1, ..., z_j) : z_i \in Z, i = 1, ..., j\}$, where Z is a finite set of possible shock realizations. $P(z^j)$ denotes the probability of history z^j.

$$E[\sum_{j=1}^{J} \beta^{j-1} u(c_j)] = \sum_{j=1}^{J} \sum_{z^j \in Z^j} \beta^{j-1} u(c_j(z^j))P(z^j)$$

An agent solves the decision problem below, taking initial financial wealth $k_1(1+r)$, initial human capital h_1 and learning ability a as given.

$$\max E[\sum_{j=1}^{J} \beta^{j-1} u(c_j)]$$

subject to

(1) $c_j + k_{j+1} = k_j(1+r) + e_j, \forall j$ and $k_{J+1} \geq 0$

(2) $e_j = R_j h_j L_j$ if $j < J_R$, and $e_j = 0$ otherwise.

(3) $h_{j+1} = z_{j+1} F(h_j, l_j, a), \forall j$

(4) $L_j + l_j = 1, \forall j$

In this decision problem the agent faces a period budget constraint in which consumption c_j plus financial asset holding k_{j+1} equal earnings plus the value of assets brought into the period. Financial assets pay a risk-free, real return r. Earnings e_j before a retirement age J_R equal the product of a human capital rental rate R_j, an agent's human capital h_j and the fraction L_j of available time put into market work. Earnings are zero at and after the retirement age. An agent's future human capital is determined (see condition (3)) by an idiosyncratic shock z_{j+1} multiplying the law of motion for human capital F. The law of motion F depends upon current human capital h_j, time devoted to human capital production l_j and an agent's learning ability a, and is increasing in its three arguments.

We now comment on three key features of the model. First, while the earnings of an agent are stochastic over the life cycle, the earnings distribution for a large cohort of agents evolves deterministically. This occurs because the model has idiosyncratic but no aggregate risk.[7] Second, the model has two sources of growth in earnings dispersion over the lifetime - agents have different learning abilities and agents have different shock realizations over the lifetime. The next section characterizes empirically the rise in US earnings dispersion for cohorts of US males. The third key feature is that the model implies that the nature of human capital shocks can be identified from wage rate data, independently from all other model parameters. This holds (as an approximation) towards the end of the working life cycle because optimal human capital investment goes to zero. The next section develops the logic of this point and estimates parameters of the shock process from the relevant moments of wage rate data.

3 Data and Empirical Analysis

In this section we use US data to address two issues. First, we characterize how mean earnings and a measure of earnings dispersion evolve with age for a cohort of individuals. Second, we estimate a process for human capital shocks by using wage rate data.

3.1 Age Profiles

The age profiles are based on earnings data from the PSID 1969-2004 family files. We utilize earnings of males who are the head of the household, who work between 520 and 5820 hours per year and who earn at least 2000 dollars (in 1968 prices). We consider males between the ages of 21 and 62. These selection criteria are motivated by several considerations. First, the PSID has many observations in the middle but relatively fewer at the beginning or end of the working life cycle. By focusing on ages 21-62, we have at least 100 observations in each age-year bin with which to calculate age and year-specific earnings statistics. Our age bins are centered 5 year age bins. For each year we therefore have bins for ages 23- 60. Second, near the traditional retirement age there is a substantial fall in labor force participation that occurs for reasons that are abstracted from in the model we analyze. This suggests the use of a terminal age that is earlier than

[7] More specifically, $P(z^j)$ is both the probability that an agent receives a j-period shock history z^j and the fraction of the agents in a cohort that receive this shock history.

the traditional retirement age.

Let $e_{j,t}$ be the mean real earnings of agents who are age j at time t.[8] The earnings data can be viewed as being generated by several factors, that we name as cohort, time, and age effects. Ultimately, we are interested in the age effect. However, as described in detail below, this measure depends on the identifying assumptions regarding cohort and time effects. To introduce notation we denote a birth cohort as $s = t - j$ that is agents who were born in year $t - j$. We assume that $e_{j,t}$ is determined by cohort effects α_s, age effects β_j, time effects γ_t and shocks $\epsilon_{j,t}$. The relationship between these variables is given below both in levels and in logs, where the latter is denoted by a tilde. Cohort effects can be viewed as effects that are common to all agents who were born in a particular year (e.g., those who were born in the Great Depression may have suffered a permanent adverse shock). Time effects can be viewed as effects that are common to all individuals alive at a point in time. An example would be a temporary rise in the rental rate of human capital that increases the earnings of all individuals in the period.

$$e_{j,t} = \alpha_s \beta_j \gamma_t \epsilon_{j,t} \tag{1}$$

$$\tilde{e}_{j,t} = \tilde{\alpha}_s + \tilde{\beta}_j + \tilde{\gamma}_t + \tilde{\epsilon}_{j,t} \tag{2}$$

The linear relationship between time t, age j, and birth cohort $s = t - j$ limits the applicability of this regression specification. Specifically, without further restrictions the regressors in this system are co-linear and these effects cannot be estimated. This identification problem is well known.[9] In effect any trend in the data can be arbitrarily reinterpreted as due to year (time) effects or alternatively as due to age or cohort effects.

Given this problem, we provide two alternative measures of the age effects. These correspond to the cohort effects case where we set $\tilde{\gamma}_t = 0,\ \forall t$ and the time effects case where we set $\tilde{\alpha}_s = 0,\ \forall s$. We use ordinary least squares to estimate the coefficients. For the cohort effects case, the regression has $J \times T$ dependent variables regressed on $J + T$

[8] Real values are calculated using the CPI. To calculate $e_{j,t}$ we use a 5 year bin centered at age j. For example, to calculate mean earnings of agents age $j = 30$ in year $t = 1980$ we use data on agents age $28 - 32$ in 1980.

[9] See Weiss and Lillard (1978), Hanoch and Honig (1985) and Deaton and Paxson (1994) among others.

cohort dummies and J age dummies. T and J denote the number of time periods in the panel and the number of distinct age groups, which in our case equal $J = 60 - 23$ and $T = 2004 - 1969$. For the time effects case the regression has $J \times T$ dependent variables regressed on T time dummies and J age dummies. This regression has J less regressors than the regression incorporating cohort effects.

In Figure 1 we graph the age effects of the *levels* of earnings implied by each regression. Figure 1 highlights the familiar hump-shaped profile of mean earnings. Figure 1 is constructed by plotting β_j from each regression above. The age effects β_j are scaled so that mean earnings equal 100 at the end of the working life cycle.

[Insert Figure 1]

A similar analysis can be carried out in order to extract the age profile of measures of earnings dispersion. We consider two standard measures of dispersion: the variance of log earnings and the Gini coefficient of earnings. Specifically, let $v_{j,t}$ and $g_{j,t}$ respectively be the cross-sectional variance of log earnings and the Gini coefficient of those agents who are of age j in year t.[10] Then the age profiles are derived from the following regressions.

$$v_{j,t} = \alpha_s^v + \beta_j^v + \gamma_t^v + \epsilon_{j,t}^v \tag{3}$$

$$g_{j,t} = \alpha_s^g + \beta_j^g + \gamma_t^g + \epsilon_{j,t}^g. \tag{4}$$

Figure 2 and 3 provide the age effects based on cohort and time effects for each measure of earnings dispersion. Again, the cohort effects are derived by setting $\gamma_t = 0,\ \forall t$, while the time effects are constructed by setting $\alpha_s = 0,\ \forall s$ for both the log variance and Gini regressions. Figure 2 plots the variance of log earnings while Figure 3 plots the age effects for the Gini measure. In each case, the solid line corresponds to cohort effects whereas the dashed line corresponds to time effects. In Figure 2 we see that the cohort effect view implies a rise of about 0.4 from age 23 to 60 while the time effects imply a smaller rise in

[10] More specifically, we use 5 year age bins centered at age j to compute these statistics, which is the same construction as for the analysis of mean earnings.

variance of log earnings of only about 0.2. The same qualitative pattern can be seen in Figure 3.

[Insert Figures 2 and 3]

3.2 Human Capital Shocks

The model implies that an agent's wage rate, measured as market compensation per unit of work time, is precisely equal to the product of the rental rate and an agent's human capital. The model also implies that late in the working life cycle human capital investments are approximately zero. This occurs as the number of working periods over which the agent can reap the returns to these investments falls as the agent approaches retirement. The upshot is that when there is no human capital investment over a period of time, then the change in an agent's wage rate is entirely determined by rental rates and the human capital shock process and not by any other model parameters.[11]

This logic is restated in the equations below. The first equation indicates how the wage w_{t+s} is determined by rental rates R_{t+s} and shocks z_{t+s} in the absence of human capital investment. Here it is assumed that there is no human capital investment from period t to $t+s$ so that $F(h,0,a) = h$ in all periods with no investment. The second equation takes logs of the first equation, where a hat denotes a log of a variable.

$$w_{t+s} \equiv R_{t+s}h_{t+s} = R_{t+s}z_{t+s}F(h_{t+s-1},0,a) = R_{t+s}z_{t+s} \times ... \times z_{t+1}h_t$$

$$\hat{w}_{t+s} \equiv \log w_{t+s} = \hat{R}_{t+s} + \sum_{j=1}^{s} \hat{z}_{t+i} + \hat{h}_t$$

Now let measured s-period log wage differences (denoted $y_{t,s}$) be true differences plus measurement error differences. This is the first equation below. This equation then implies that the three cross-sectional moment conditions below hold. These moments are based on the assumption that human capital shocks $\hat{z}_t$ are independent across time and people, that $\hat{z}_t \sim N(\mu, \sigma^2)$ and that measurement errors ϵ_t are independent and identically distributed

[11]Heckman et al (1998) use this line of reasoning to estimate differences in rental rates across skill groups within a model which abstracts from the type of idiosyncratic risk that we focus on.

across time and people with variance $Var(\epsilon_t) = \sigma_\epsilon^2$. We also assume that measurement errors and human capital shocks are jointly independent.

$$y_{t,s} \equiv \hat{w}_{t+s} - \hat{w}_t + \epsilon_{t+s} - \epsilon_t = \hat{R}_{t+s} - \hat{R}_t + \sum_{j=1}^{s} \hat{z}_{t+i} + \epsilon_{t+s} - \epsilon_t$$

$$E[y_{t,s}] = \hat{R}_{t+s} - \hat{R}_t + s\mu \quad (5)$$

$$Var(y_{t,s}) = s\sigma^2 + 2\sigma_\epsilon^2 \quad (6)$$

$$Cov(y_{t,s}, y_{t,r}) = r\sigma^2 + \sigma_\epsilon^2 \text{ for } r < s \quad (7)$$

To make use of these moment restrictions, one needs to be able to measure the variable $y_{t,s}$ and to have agents for which the assumption of no time spent learning is a reasonable approximation. The focus on older workers solves both issues. Wage data for younger workers are potentially problematic for both issues. Specifically, on the first issue it may be difficult to accurately measure the wage rates emphasized in the model when measured time at work is a mix of work time and learning time.

We use wages defined as total labor earnings divided by total hours for male head of household. We impose the same selection criteria as those presented in Section 3.1 for earnings. We follow males in the PSID for either three years or four years. Thus, we calculate two log wage differences (i.e. $y_{t,s}$ for $s = 1, 2$) when males are followed for three years and three log wage differences when males are followed for four years.[12] In estimation we use cross sectional moments aggregated across panel years. That is, for each year we generate the sample analog to the moments in equation (**??**)-(**??**), that is $\mu_{t,s} \equiv \frac{1}{N_t}\sum_{i=1}^{N_t} y_{t,s}^i$ and $\frac{1}{N_t}\sum_{i=1}^{N_t}(y_{t,s}^i - \mu_{t,s})^2$ and $\frac{1}{N_t}\sum_{i=1}^{N_t}(y_{t,s}^i - \mu_{t,s})(y_{t,r}^i - \mu_{t,r})$. We stack the moments across the panel years and use a 2-step GMM estimation with an identity matrix as the initial weighting matrix.

Table 1 provides the estimation results for various data selections. Over the entire sample period the point estimate of the standard deviation σ of the shock to human

[12] The PSID data is not available for the years 1997, 1999, 2001, and 2003. In the years preceding those years we impose that the agent is available for three consecutive years and use a two year growth rate.

Table 1: Estimation of Human Capital Shocks

Min-Age	Max-Age	Period	N	σ	S.E.(σ)	σ_ϵ	S.E.(σ_ϵ)	$\bar{s}$
55	65	1969-2004	125	0.108	(0.029)	0.153	(0.013)	2
50	60	1969-2004	223	0.110	(0.023)	0.157	(0.011)	2
23	60	1969-2004	1521	0.158	(0.010)	0.177	(0.006)	2
55	65	1969-2004	106	0.103	(0.023)	0.149	(0.012)	3
50	60	1969-2004	200	0.104	(0.019)	0.151	(0.010)	3
23	60	1969-2004	1406	0.140	(0.009)	0.178	(0.005)	3
55	65	1969-1981	119	0.088	(0.040)	0.150	(0.015)	2
50	60	1969-1981	225	0.105	(0.027)	0.146	(0.013)	2
23	60	1969-1981	1322	0.152	(0.013)	0.166	(0.008)	2

The entries provide the estimates for σ and σ_ϵ for various samples. The first and second column provide the minimum and maximum respective age in the sample. The third column refers to which PSID years are included. The column labeled N refers to the median number of observation across panel years. Columns labeled *S.E.* refer to standard errors. The column denoted $\bar{s}$ refers to the maximum s value used in computing log wage differences. In estimation all variance and covariance restrictions are always imposed.

capital is between 0.10 and 0.11 for both the age group $50 - 60$ and $55 - 65$. This holds when we follow males for three years ($\bar{s} = 2$) or for four years ($\bar{s} = 3$). This is smaller than the estimate which obtains when all males $23 - 60$ are pooled together. Recall from the point of view of human capital theory that including younger age groups will mean that the change in log wages will be determined both by shocks and endogenous human capital decisions rather than by shocks alone. We note that the magnitude of the persistent wage shocks estimated by Heathcote et al (2005) in PSID data when various age groups are pooled is between our estimates for the older age groups and our estimate for the pooled sample. Finally, we note that the lower part of Table 1 contains results for when only the early part of the sample period is used in estimation. Here we find that the point estimates of the shocks are lower for each age group than the estimates obtained using the entire time period 1969- 2004. Over the early part of the sample period we find that the estimated shocks for the 50-60 and the 55-65 age groups are smaller than those estimated when all age groups are pooled. This is the same pattern as was found over the entire sample period.

4 Setting Model Parameters

The strategy for setting model parameters is in three steps. First, we estimate the parameters governing human capital shocks directly. This was done in the previous section. Second, we choose parameters governing the utility function, interest rates and the human capital production function based upon previous studies. Third, we set the parameters governing the distribution of initial conditions so that the model best matches the age profiles of mean earnings and earnings dispersion from the previous section. In choosing this initial distribution, we take all other model parameters as given.

Model parameter values are summarized in Table 2. The top panel in Table 2 highlights parameters that we set based upon other studies. We set the model period to be a year. Agents live $J = 56$ model periods or from a real-life age of 20 to 75. We set a retirement age at $J_R = 42$ or at a real-life age of 61. At the retirement period an agent can no longer engage in market work. The real interest rate in the model is set to $r = 0.042$ and equals the average of the annual return to stock and long-term bonds over the period 1946-2001 (see Siegel (2002, Table 1-1 and 1-2)). The discount factor is set to $\beta = 1/(1 + r)$ so that absent risk the consumption profile solving the model is flat.

The utility function is of the constant relative risk aversion class. The parameter ρ governing risk aversion and intertemporal substitution is set to $\rho = 2$. This value is around the middle of the estimates of this parameter from micro-level data which are surveyed by Browning, Hansen and Heckman (1999, Table 3.1). The law of motion for human capital embodies the Ben-Porath (1967) human capital production function. The human capital literature has estimated the elasticity parameter α governing the production of new human capital to lie in the range 0.5 to just over 0.9. These estimates are surveyed by Browning et. al. (1999, Table 2.3- 2.4). We set α to lie in the middle of this range.

Table 2: Parameter Values

Definition	Symbol	Value
Model Periods	J	$J = 56$
Retirement Period	J_R	$J_R = 42$
Interest Rate	r	$r = 0.042$
Discount Factor	β	$\beta = 1.0/(1 + r)$
Preferences	$u(c)$	$u(c) = \frac{c^{(1-\rho)}}{(1-\rho)}$ $\rho = 2$
Law of Motion for Human Capital	$F(h, l, a)$	$F(h, l, a) = h + a(hl)^{\alpha}$ $\alpha = 0.7$
Rental Rate	R_j	$R_j = (1 + g)^{j-1}$ $g = 0$
Human Capital Shocks	z	$\log(z) \sim N(\mu, \sigma^2)$ $\sigma = 0.088 - 0.108$ $\mu = -0.0284$ to $-yy$
Distribution of Initial Conditions	ψ	$x \equiv (h_1, k_1, a) \sim \psi$ discussed below

The setting of parameter values in the lower panel of Table 2 is described next. First, we set the growth rate g of the rental rate of human capital so that the model accounts for growth in average earnings in US data. [This is NOT currently done.] The model predicts, due to homothetic preferences, that the earnings distribution of different cohorts is shifted proportionally by the initial level of the rental rate of human capital, holding the initial distribution of human capital and learning ability equal. This would also imply that average cross-sectional earnings in the model grow over time at rate g. Therefore, we set $g = xx$ to equal the average real growth rate in cross-sectional earnings of males in PSID data from 1969- 2004. Second, we set the standard deviation σ of the log human capital shocks to be consistent with the estimates in Table 1. We analyze the case when $\sigma = 0.088$ and when $\sigma = 0.108$, which are respectively the highest and the lowest point esitmates from Table 1 based upon the $55-65$ age group. Third, we set μ governing the mean log human capital shock so that the model matches the average rate of decline of mean earnings for the cohorts of older workers in US data that we documented earlier in Figure 1. The fall in mean earnings in the model equals $(1+g)e^{\mu+\sigma^2/2}$ when agents make no human capital investments. Thus, μ is set, given the value g and σ, so that this holds.

We restrict the initial distribution of human capital, assets and learning ability to lie in a parametric class. In the benchmark model, we set initial assets to zero and specify that initial human capital and learning ability are jointly lognormally distributed so that $\log(x) \sim N(\mu_x, \Sigma)$, when $x = (h_1, a)$. We then choose (μ_x, Σ) to best match the dynamics of the US earnings distribution documented in section 3, given all other model parameters. The next section highlights properties of the resulting initial distributions. Later in the paper we explore a trivariate distribution of initial conditions where the initial asset distribution matches features of the (scaled) empirical values for net wealth for young households in the PSID.

5 Earnings in the Model

In this section, we report on the ability of the model to reproduce the earnings facts we documented in section 3. We first show the model implications for the evolution of the earnings distribution for a cohort. We then assess the role of learning ability and human capital risk in generating these properties. We focus on the case in which all individuals start their life cycle with zero assets.

5.1 Dynamics of the Earnings Distribution

The age profiles of mean earnings and earnings dispersion produced by the benchmark model are displayed in Figure 4 below. The model generates the hump-shaped earnings profile for a cohort by a standard human capital accumulation argument. Early in the life cycle, the bulk of individuals accumulate human capital in net terms and progressively devote increasing fractions of their time to market work. Both effects act to increase mean earnings as agents age. Towards the end of the working life-cycle, mean human capital accumulation for a cohort levels off, and eventually falls. We note that human capital falls when the mean multiplicative shock to human is smaller than one (i.e. $E[z] = e^{\mu+\sigma^2/2} < 1$). This corresponds to the notion that on average human capital depreciates. The implication is that average earnings fall late in life when growth in the rental rate of human capital is not enough to offset the mean fall in human capital.

[Insert Figure 4 a-b Here]

Two forces account for the rise in earnings dispersion. First, since individual human capital is repeatedly hit by shocks, these shocks are a source of increasing dispersion in human capital and earnings as a cohort ages. Second, differences in learning ability across agents produce mean earnings profiles with different slopes. This follows since for a common level of current human capital, agents with high learning ability choose to produce more human capital than their low ability counterparts. Huggett et. al. (2006, Proposition 1) establish that this holds in the absence of human capital risk. This mechanism implies that earnings of high ability individuals are relatively low early in life, and relatively high late in life.

We now try to understand the quantitative importance of these two forces for producing the results in Figure 4 by alternatively eliminating ability differences or eliminating shocks, holding all other parameters at their previous values. To highlight the role of learning ability differences, we change the initial distribution so that all agents have the same learning ability, which we set equal to mean ability. In the process of changing learning ability in this way, we do not alter any agent's initial human capital. Figure 5 shows that eliminating ability differences leads to the striking result that the rise in earnings dispersion over the lifetime is almost completely eliminated.

The pattern of dispersion that remains after removing ability differences is due to two opposing forces which largely cancel out except towards the end of the working lifetime. First, human capital risk leads ex-ante identical agents to differ ex-post in human capital and earnings. Second, the model has a force which leads to decreasing dispersion in human capital and earnings for a cohort as the cohort ages. Without risk and without ability differences, all agents within an age group produce the same amount of new human capital regardless of the current level of human capital- see Huggett et. al. (2006, Proposition 1). This holds for the constant elasticity human capital production functions analyzed in this paper. An implication is that the distribution of human capital and earnings are Lorenz ordered by age and, thus, measures of earnings or human capital dispersion decrease for a cohort as the cohort ages. At the end of life earnings dispersion increases since human capital production goes to zero and, thus, the force leading to convergence is eliminated.

[Insert Figure 5 Here]

To highlight the role of human capital risk, we eliminate idiosyncratic risk altogether by setting $\sigma = 0$, while adjusting the mean log shock μ to keep the mean shock level constant in both cases. Figure 6 a-b illustrate that removing idiosyncratic risk leads to a counter-clockwise rotation of the mean earnings profile. This also leads to a U-shaped earnings dispersion profile.

[Insert Figure 6 a-b Here]

When idiosyncratic risk is eliminated, human capital accumulation becomes more attractive for risk-averse individuals. Thus, all else equal, individuals' response dictates lower mean earnings early in life from the greater time spent accumulating human capital and a higher growth rate in mean earnings early in life. The result is a counter-clockwise movement in the mean earnings profile.[13] In terms of dispersion in labor earnings, human capital shocks are more important for agents with relatively high learning ability. These agents are the ones who would allocate an even larger fraction of time into human capital accumulation for lower values of the variance of idiosyncratic shocks. When human capital risk is eliminated, these agents allocate less time to work early in life and more time to human capital accumulation. Consequently, earnings dispersion is higher at the start of the working life-cycle. Earnings dispersion falls for the first 10 years of the working lifetime. At this point the earnings of higher learning ability agents are overtaking the earnings of their lower ability counterparts. Subsequently, earnings dispersion increases because of the differences in slopes of age-earnings profiles across ability levels.

[13]This is effectively the central result of Lehvari and Weiss (1974) extended to a multi-period setting. They showed in a two-period model that time input into human capital production is smaller with human capital risk than without when agents are risk averse.

5.2 Properties of the Initial Distribution

What are the properties of the initial distributions that best reproduce the earnings facts? To better highlight the role of human capital risk, Table 3 shows these properties for two values of the magnitude of idiosyncratic shocks $\sigma = 0.088$ and for $\sigma = 0.108$. These values for the standard deviation of shocks were point estimates from Table 1 calculated over different time periods. In each case, we adjust μ so that the decline in mean earnings matches the observed value at the end of the life cycle.

Table 3 shows a number of properties. First, learning ability and initial human capital are positively correlated. This will have implications for how later on in the paper we go about highlighting the role of learning ability versus initial human capital as sources of lifetime inequality. Second, the distribution of initial conditions moves in a systematic way with the magnitude of shocks. As the standard deviation of shocks increases, the initial distributions that best reproduce the earnings facts require higher levels of mean initial learning ability and lower levels of ability dispersion and human capital dispersion. We will see later on that the fall in ability dispersion and human capital dispersion with increases in risk will be reflected in a reduction in how important variations in intial conditions are to variations in welfare.

Table 3: Properties of Initial Distributions

Statistic	$\sigma = 0.088$	$\sigma = 0.108$
Mean Learning Ability (a)	0.351	0.370
Coefficient of Variation (a)	0.227	0.171
Mean Initial Human Capital (h_1)	118.3	117.6
Coefficient of Variation (h_1)	0.441	0.401
Correlation (a, h_1)	0.792	0.770

What accounts for these changes in the initial distributions? To understand these changes, the reader should recall from our previous analysis that eliminating shocks from the model for a given initial distribution leads to a counter-clockwise shift in the mean earnings profile. This occurs because the time input into human capital accumulation over the life cycle increases as human capital risk decreases. This is consistent with the result of Lehvari and Weiss (1974) whereby, in a two-period model, risk-averse agents reduce human capital investments with human capital risk compared to the no risk case.

Following this intuition, to produce the earnings facts as risk increases the distribution of initial conditions needs to be adjusted. A higher mean learning ability level leads to a counter-clockwise rotation of the mean earnings profile. Thus, higher mean learning ability counteracts the clockwise rotation of the mean earnings profile produced by adding risk to the model with fixed initial distribution. A higher mean ability level has this effect as it leads to an increase in the time put into human capital production early in life. The intuition for why ability dispersion falls as human capital risk increases is straightforward. This is because human capital risk is itself a source of increased earnings dispersion over the life cycle. Thus, greater human capital risk leaves less room for ability differences in accounting for the rise in earnings dispersion with age over the life cycle.

6 Lifetime Inequality

6.1 Analysis of the Benchmark Model

We decompose the variance in lifetime inequality into variation due to initial conditions versus variation due to shocks. This is done both for lifetime utility and for lifetime wealth. Such a decomposition makes use of the fact that any random variable can be written as the sum of its condition mean plus the variation from its conditional mean. As these two components are orthogonal by construction, the total variance in the random variable equals the sum of the variance in the conditional mean plus the mean of the variance around the conditional mean.[14] We note that the variance decomposition of lifetime utility is unchanged if the expected utility function is represented by an affine transformation of the original utility function.

Table 4 decomposes lifetime inequality into the sources highlighted by the model. Each measure of lifetime inequality is analyzed as of the start of the working life cycle, which is taken to be a real-life age of 20. We focus on two cases for the magnitude of idiosyncratic

[14] Formally, let $f(x,y) = E[f(x,y)|x] + (f(x,y) - E[f(x,y)|x]) = E[f(x,y)|x] + \epsilon(x,y)$. Orthogonality then implies $var(f(x,y)) = var_x(E[f(x,y)|x]) + E_x[var_y(\epsilon(x,y))]$. In our application, x captures initial conditions (i.e. human capital, learning ability and financial wealth) and y captures shock histories.

shocks: a high shock case $\sigma = 0.108$ as well as a low shock case $\sigma = 0.088$. Table 4 shows that for each case the majority of the variation in lifetime utility or lifetime wealth is due to variation in initial conditions.[15] Specifically, from 63 to 75 percent of the variation in lifetime utility and from 59 to 73 percent of the variation in lifetime wealth is due to initial conditions.

Table 4: Sources of Lifetime Inequality

Statistic	$\sigma = 0.088$	$\sigma = 0.108$
Fraction of Variance in Lifetime Utility Due to Initial Conditions	0.754	0.627
Fraction of Variance in Lifetime Wealth Due to Initial Conditions	0.729	0.589

The analysis so far has not addressed how important variation in one type of initial condition is compared to variation in other types for how an agent fares in life. One possible answer would involve decomposing the variance of expected lifetime utility and expected lifetime wealth. This is not likely to be very helpful as initial human capital and learning ability, the two initial conditions considered within the benchmark model, are strongly correlated. This was documented earlier in Table 3. We therefore proceed in a different way. First, we ask the agents in the model how much compensation is equivalent to starting life with a one standard deviation change in any initial condition. We express this compensation, which we call an equivalent variation, in terms of the percentage change in consumption in all periods that would be required to leave an agent with given initial condition with the same expected lifetime utility as an agent with a one standard deviation change in the relevant initial condition. Second, we determine the percent by which an agent's expected lifetime earnings will change in response to a one standard deviation change in any initial condition. In all cases, the baseline initial

[15] We define lifetime utility and lifetime wealth along a given lifetime shock history z^J as follows: $U(z^J; h_1, k_1, a) = \sum_{j=1}^{J} \beta^{j-1} u(c_j(z^J; h_1, k_1, a))$ and $W(z^J; h_1, k_1, a) = k_1(1 + r) + \sum_{j=1}^{J} e_j(z^J; h_1, k_1, a)/(1 + r)^{j-1}$.

condition is set so that $(\log h, \log a)$ equals the mean log values of human capital and learning ability in the baseline model and the changes are also in log standard deviations.

Table 5: The Importance of Changes in Initial Conditions

Equivalent Variations			
Variable	Change in Variable	$\sigma = 0.088$	$\sigma = 0.108$
Learning Ability	+ 1 st. deviation	9.6%	5.6%
	− 1 st. deviation	−5.5%	−5.1%
Human Capital	+ 1 st. deviation	37.2%	31.5%
	− 1 st. deviation	−27.5%	−24.7%
Expected Lifetime Earnings			
Variable	Change in Variable	$\sigma = 0.088$	$\sigma = 0.108$
Learning Ability	+ 1 st. deviation	11.8%	7.5%
	− 1 st. deviation	−6.6%	−6.4%
Human Capital	+ 1 st. deviation	37.6%	31.3%
	− 1 st. deviation	−27.7%	−23.0%

The top panel of Table 5 states equivalent variations associated with changes in each variable. The bottom panel of Table 5 states the percentage change in the expected present value of earnings associated with changes in each variable. The 1 standard deviation movements are in logs.

Table 5 presents the results of this analysis. Here we find that a one standard deviation movement in either the log of initial human capital or the log of learning ability both give rise to substantial changes in welfare as measured by equivalent variations. A one standard deviation increase in initial human capital is equivalent to a 31 percent increase in consumption when the standard deviation in the shock equals $\sigma = 0.108$. A one standard deviation decrease in human capital leads to a 24 percent decline in consumption. On the other hand, a one standard deviation increase or decrease in learning ability is equivalent to approximately a 5 to 6 percent change in consumption. We note that when the magnitude of shocks are smaller (i.e. for the case when $\sigma = 0.088$) the importance of one standard deviation movements in human capital or ability on equivalent variations increases. This is consistent with the results in Table 3 which showed the distribution of initial human capital or learning ability displayed more dispersion when human capital shocks are smaller. We note that in all cases the impact of these changes in initial conditions on the expected present value of earnings displays a very similar pattern to those established for equivalent variations.

6.2 Sensitivity to Social Insurance

One might conjecture that the relative importance of shocks over the life cycle versus initial conditions for lifetime inequality might be sensitive to abstracting from the structure of the social insurance system. The structure of taxation has long been seen as a potentially important determinant of the level of inequality. But it is not clear if the tax system serves to reduce inequality by mainly by insuring initial conditions or mainly by insuring shocks over the lifetime.

As a first pass at the issue of the importance of social insurance we add a stylized social insurance system to the model. The social insurance system consists of a social security system and income tax system. The social security system features a proportional earnings tax together with a common retirement benefit paid to all agents after the retirement age. We set the earnings tax rate to 10.6 percent of earnings. This is the rate used to fund old age and survivors benefit component of the US social security system. We set the benefit to equal 45 percent of mean earnings in the last period of the working lifetime in the model. This is approximately the replacement rate implied by the US social security benefit function for an individual who always receives mean earnings in each year- see Huggett and Parra (2006). We view the model social security system as a rough approximation to the relationship between taxes and benefits in the US social security system, albeit one which likely overestimates the degree of progressivity built into the US system.

The income tax system in the model is based on data for effective average tax rates in the US. We use tabulations from CBO (2004, Table 3A and 4A) for the tax year 2001 which summarize the empirical relationship between household income and average federal tax rates paid. These tabulations show that average tax rates tend to increase with income both for households with a head younger than age 65 as well as with a head older than age 65. We use the quadratic function passing through the origin that minimizes the squared deviations of the resulting tax function from data as the basis for the model income tax function. There is a separate tax function applicable before and after the retirement age. We follow Huggett and Parra (2006) in implementing this tax function. For this new model we then choose initial conditions to best match the earnings distribution facts highlighted earlier, holding all other model parameters at the previous values in Table 2.

Table 6: Sources of Lifetime Inequality: Model with Social Insurance

Statistic	$\sigma = 0.088$	$\sigma = 0.108$
Fraction of Variance in Lifetime Utility Due to Initial Conditions	0.682	0.60
Fraction of Variance in Lifetime Wealth Due to Initial Conditions	0.721	0.575
Fraction of Variance in Lifetime After-Tax Wealth Due to Initial Conditions	0.740	0.593

Table 6 presents an analysis of the sources of lifetime inequality in the model with social insurance. The results are that the relative importance of initial conditions versus shocks is broadly similar with or without a stylized social insurance system. This follows from comparing the entries in Table 6 to the corresponding entries in Table 4. What does differ across the models with and without social insurance is the level of inequality in lifetime after-tax wealth. The variance in this measure falls with social insurance. Thus, social insurance serves to reduce variability in this measure but does not change markedly the relative importance of shocks versus initial conditions.

References

Baker, M. (1997), Growth-rate Heterogeneity and the Covariance Structure of Life Cycle Earnings, *Journal of Labor Economics,* 15, 338-75.

Ben-Porath, Y. (1967), The Production of Human Capital and the Life Cycle of Earnings, *Journal of Political Economy,* 75, 352-65.

Bowlus, A. and J.M. Robin (2003), Twenty Years of Rising Inequality in Lifetime Labor Income Values, University of Western Ontario, manuscript.

Browning, M., Hansen, L. and J. Heckman (1999), Micro Data and General Equilibrium Models, in *Handbook of Macroeconomics,* ed. J.B. Taylor and M. Woodford, (Elsevier Science B.V, Amsterdam).

Card, D. (1999), The Causal Effect of Education on Earnings, In Orley Ashenfelter and David Card, editors, *Handbook of Labor Economics,* Volume 3, (Elsevier, Amsterdam).

Congressional Budget Office (2004), Effective Federal Tax Rates 1979- 2001, available at http://www.cbo.gov/.

Deaton, A. and C. Paxson (1994), Intertemporal Choice and Inequality, *Journal of Political Economy,* 102, 437-67.

Eaton, J. and H. Rosen (1980), Taxation, Human Capital and Uncertainty, *American Economic Review,* 70, 705- 15.

Guvenen, F. (2005), Learning your Earning: Are Labor Income Shocks Really That Persistent?, manuscript.

Heathcote, J., Storresletten, K. and G. Violante (2005a), Two Views on Inequality over the Life Cycle, *Journal of the European Economic Association: Papers and Proceedings,* 765- 75.

Heathcote, J., Storresletten, K. and G. Violante (2005b), The Cross-Sectional Implications of Rising Wage Inequality in the United States, manuscript.

Heckman, J., Lochner, L. and C. Taber (1998), Explaining Rising Wage Inequality: Explorations with a Dynamic General Equilibrium Model of Labor Earnings with Heterogeneous Agents, *Review of Economic Dynamics,* 1, 1-58.

Huggett, M., Ventura, G. and A. Yaron (2006), Human Capital and Earnings Distribution Dynamics, *Journal of Monetary Economics,* 53, 265- 90.

Huggett, M and J.C. Parra (2006), How Well Does the US Social Insurance System Provide Social Insurance?, manuscript.

Keane, M. and K. Wolpin (1997), The Career Decisions of Young Men, *Journal of Political Economy*, 105(3), 473-522.

Krebs, T (2004), Human Capital Risk and Economic Growth, *Quarterly Journal of Economics,* 118, 709- 744.

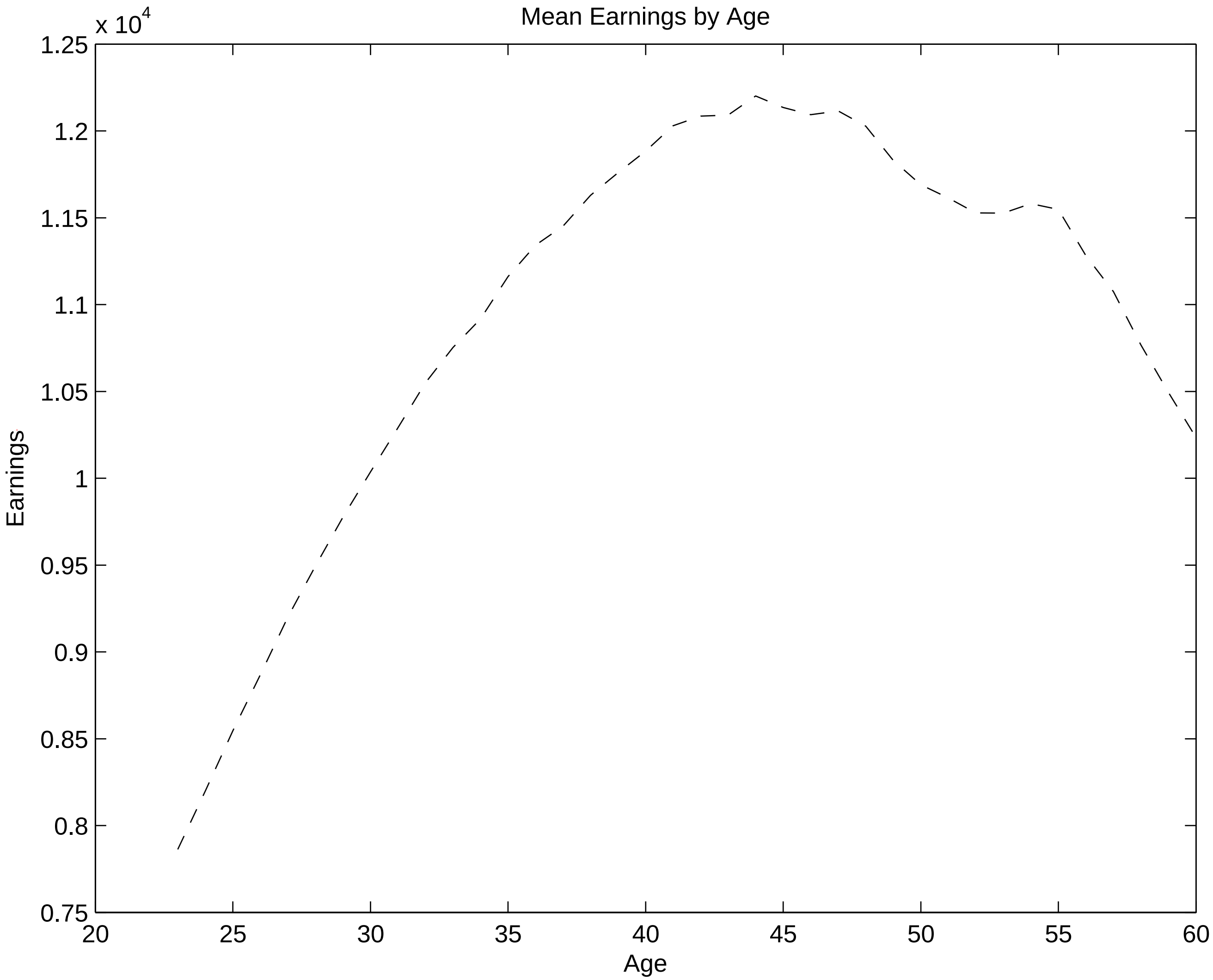
Levhari, D. and Y. Weiss (1974), The Effect of Risk on the Investment in Human Capital, *American Economic Review,* 64, 950-63.

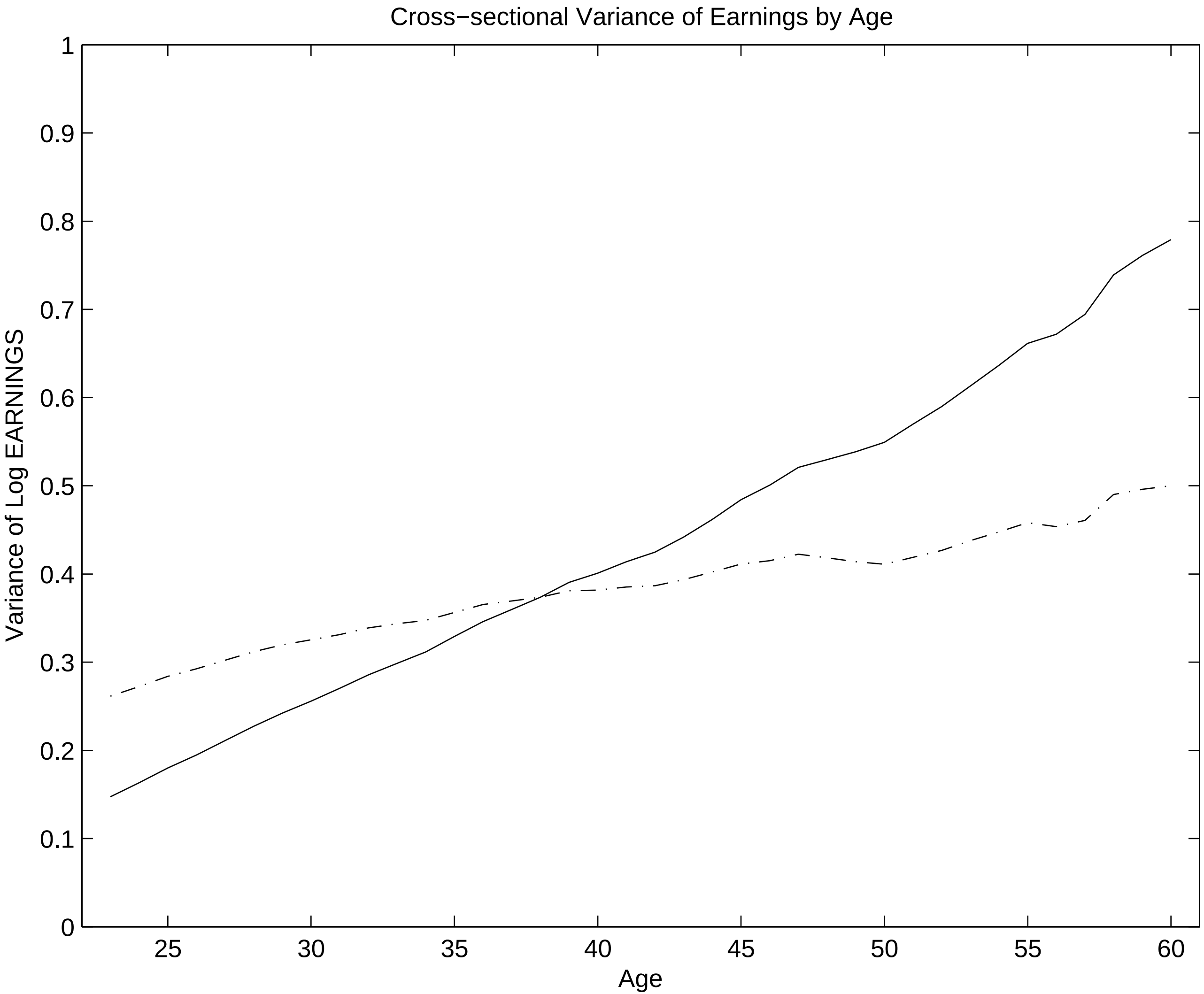
Mincer, J. (1974), *Schooling, Experience and Earnings,* Columbia University Press, New York.

Press, W. et. al. (1992), *Numerical recipes in FORTRAN,* Second Edition, (Cambridge University Press, Cambridge).

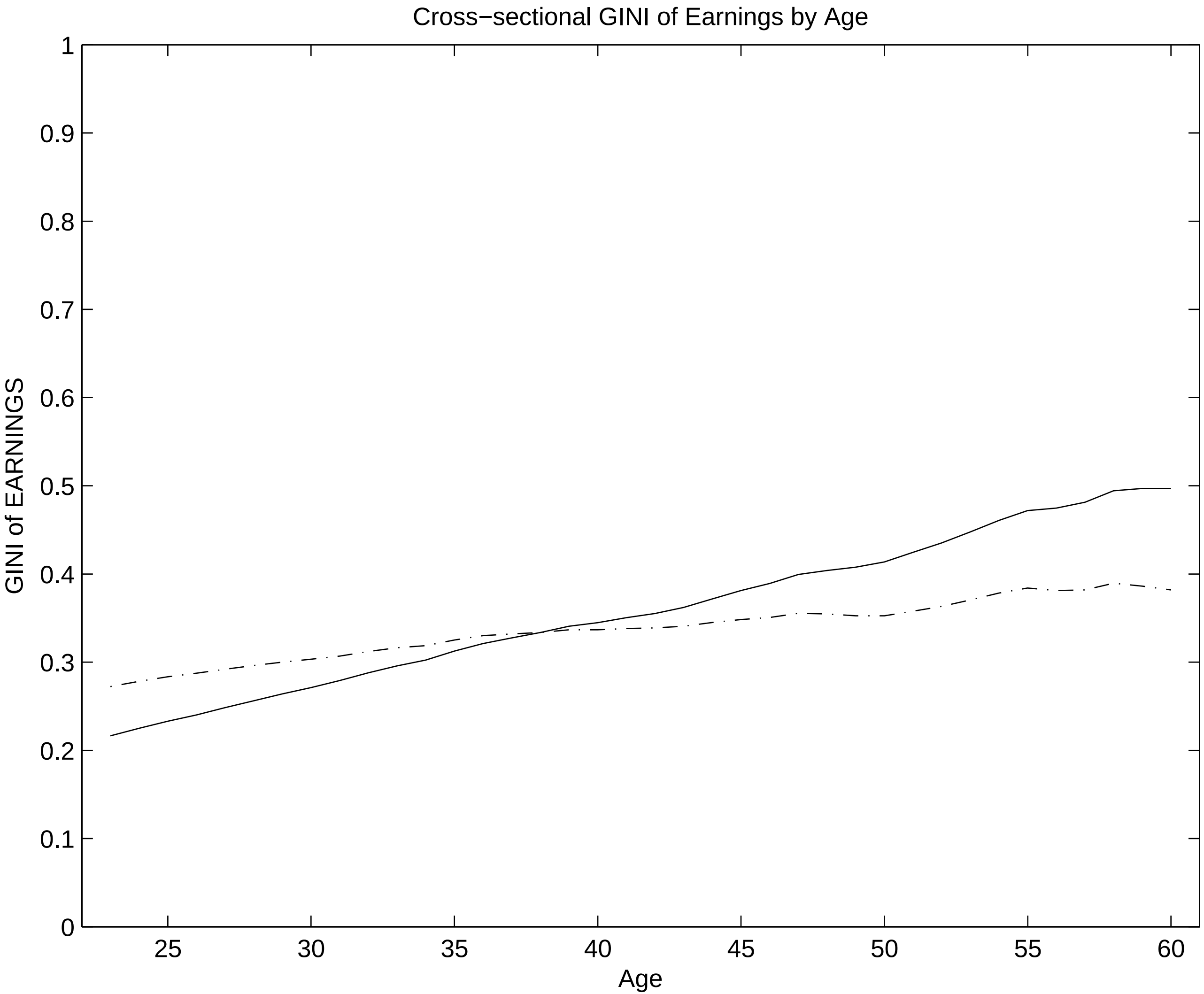
Slesnick, D. and A. Ulker (2005), Inequality and the Life Cycle: Age, Cohort Effects and Consumption, manuscript.

Storesletten, K., Telmer, C. and A. Yaron (2004), Consumption and Risk Sharing Over the Life Cycle, *Journal of Monetary Economics,* 51, 609- 33.


Mean Earnings by Age
x 10^4
1.25
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0.8
0.75
Earnings
20
25
30
35
40
45
50
55
60
Age



Cross−sectional Variance of Earnings by Age
Variance of Log EARNINGS
Age
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
25
30
35
40
45
50
55
60



Cross−sectional GINI of Earnings by Age
GINI of EARNINGS
Age
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
25
30
35
40
45
50
55
60

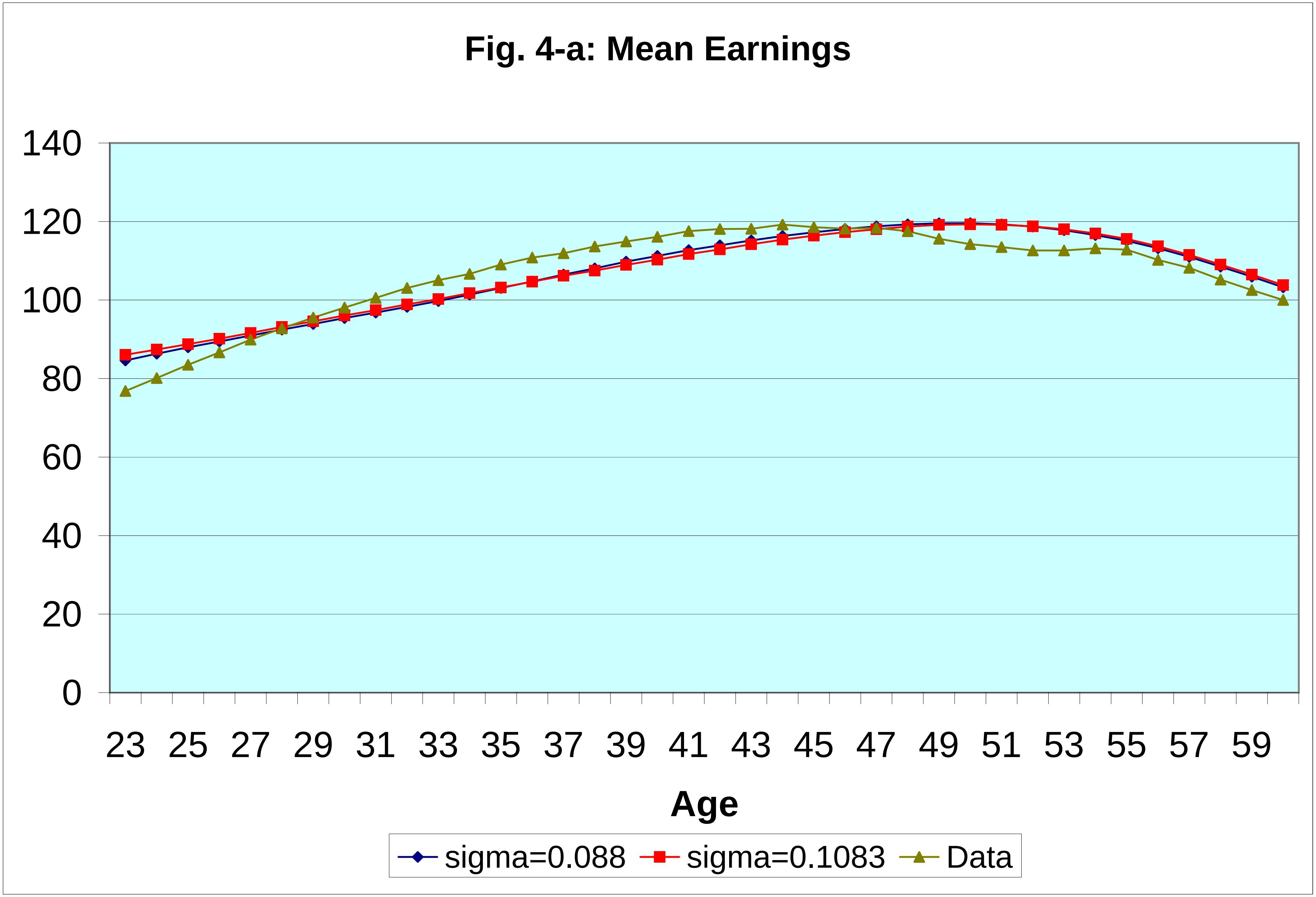


Fig. 4-a: Mean Earnings
140
120
100
80
60
40
20
0
23 25 27 29 31 33 35 37 39 41 43 45 47 49 51 53 55 57 59
Age
sigma=0.088
sigma=0.1083
Data

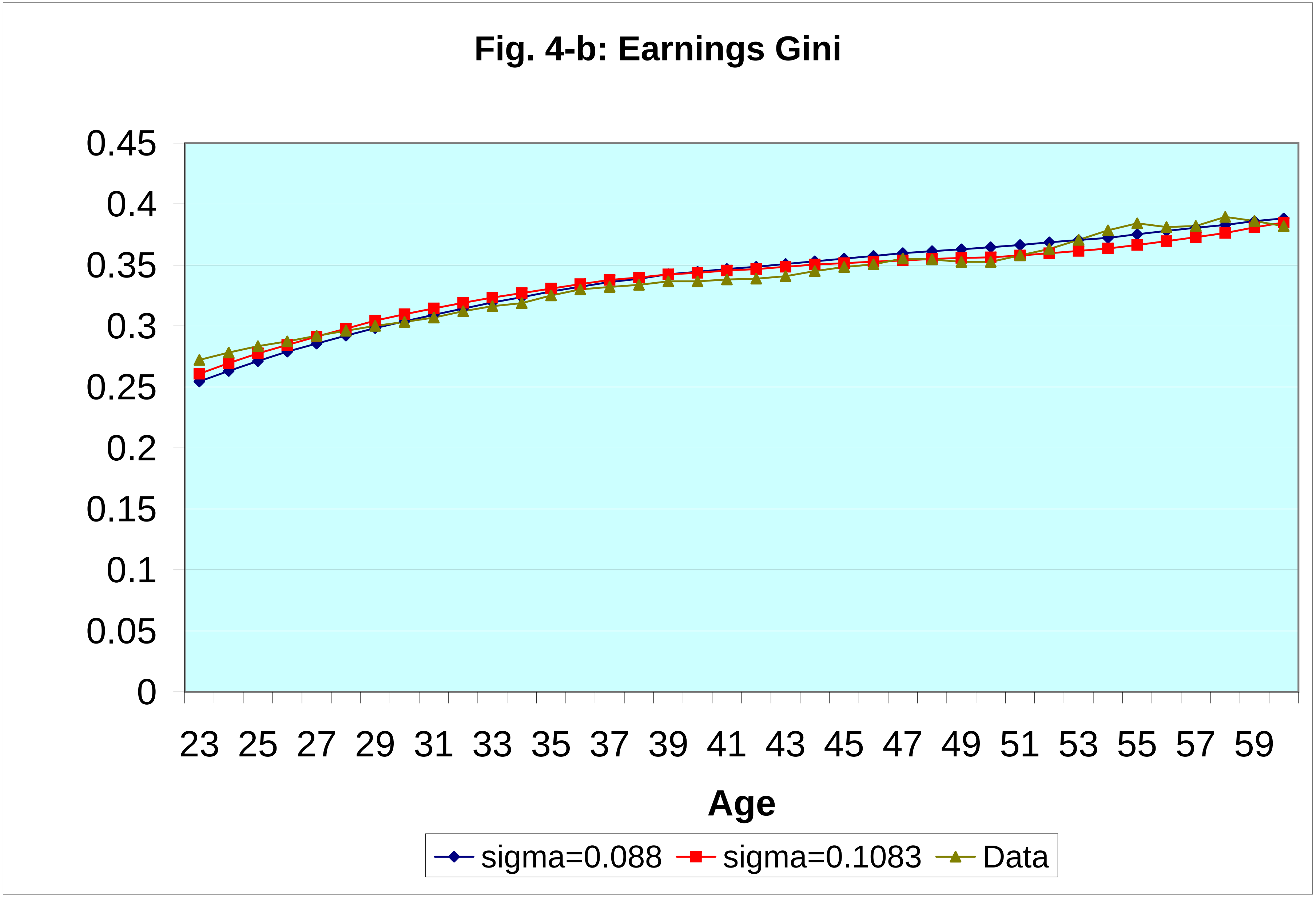


Fig. 4-b: Earnings Gini
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
23 25 27 29 31 33 35 37 39 41 43 45 47 49 51 53 55 57 59
Age
sigma=0.088
sigma=0.1083
Data

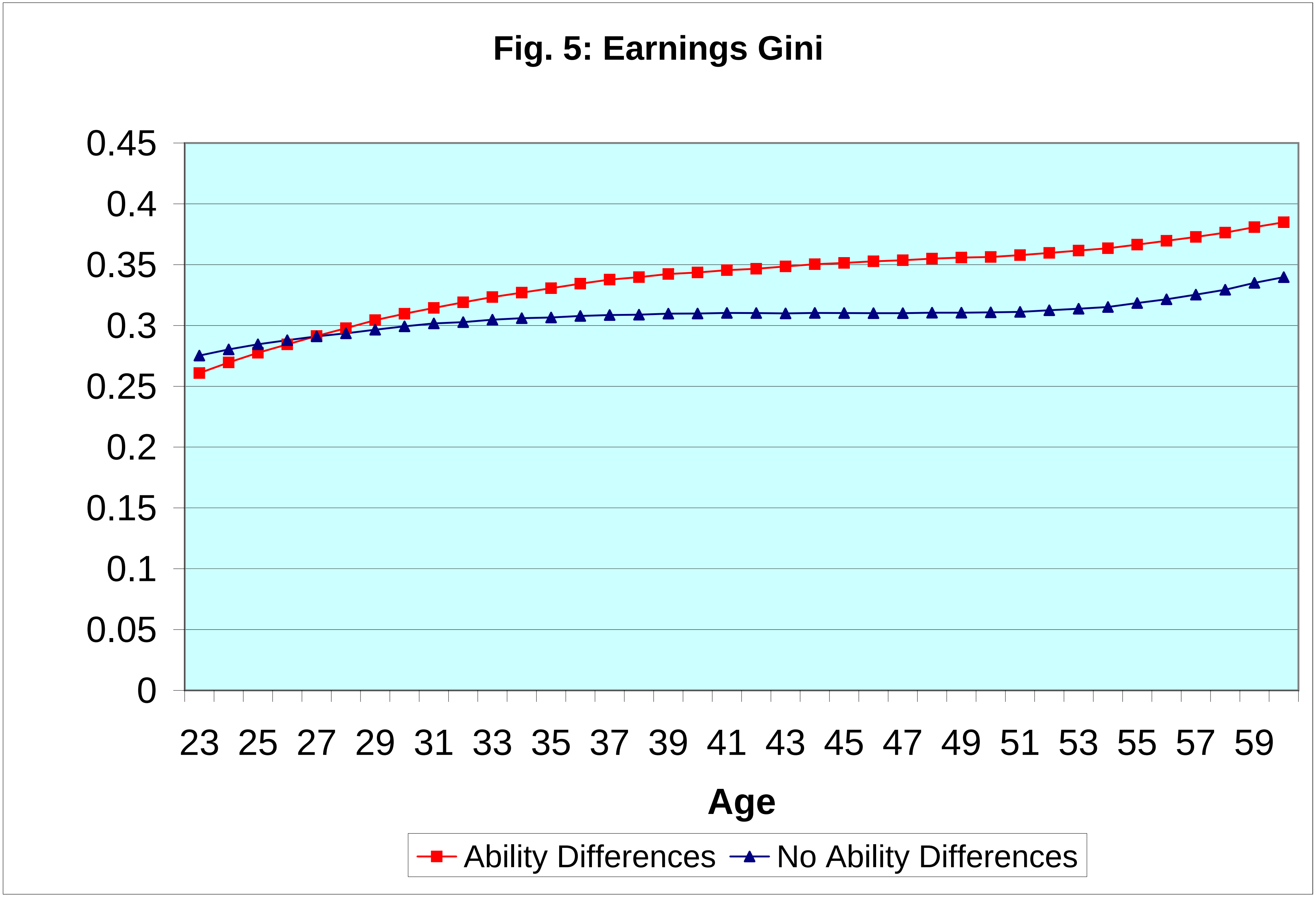


Fig. 5: Earnings Gini
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
23 25 27 29 31 33 35 37 39 41 43 45 47 49 51 53 55 57 59
Age
Ability Differences
No Ability Differences

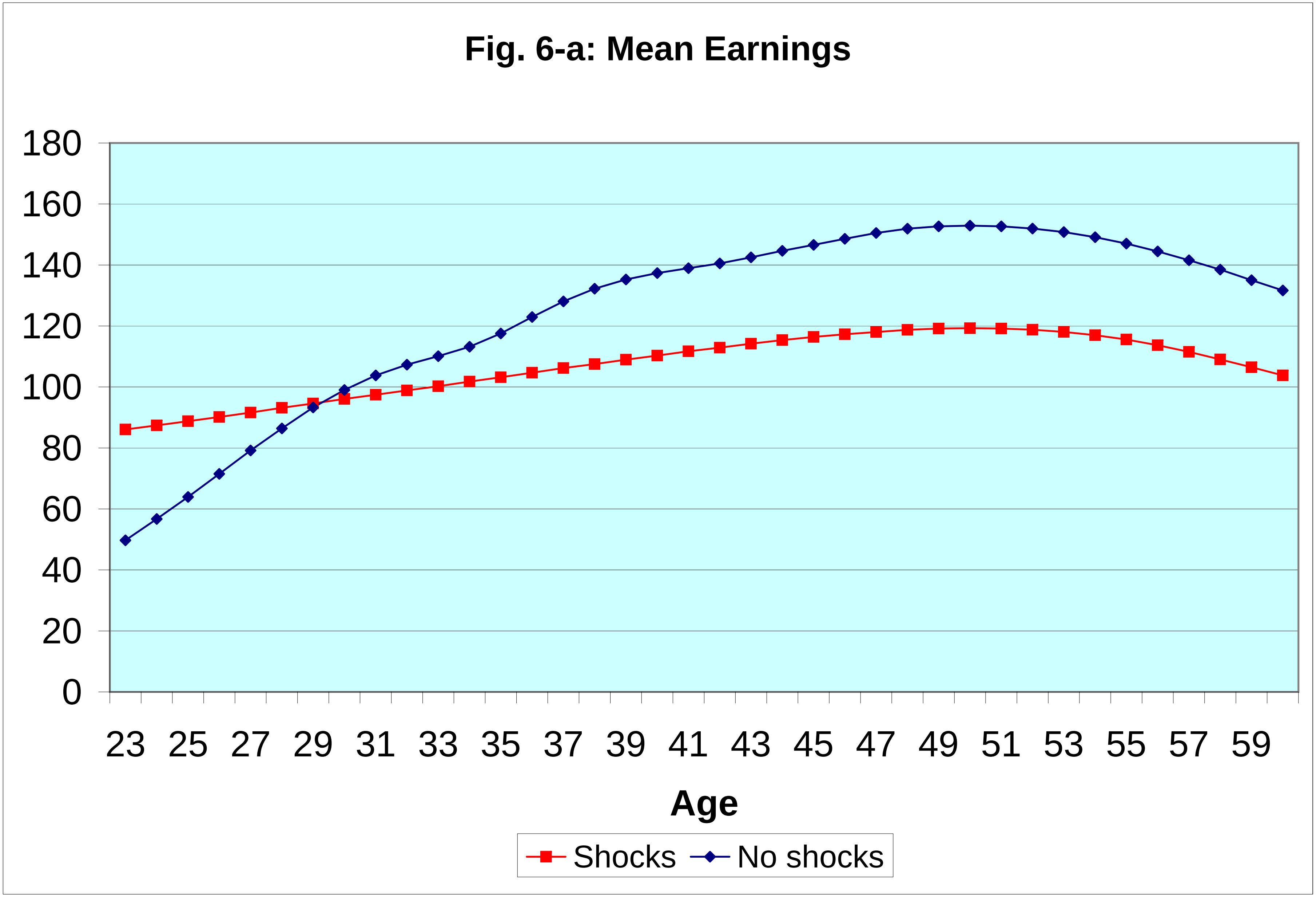


Fig. 6-a: Mean Earnings
180
160
140
120
100
80
60
40
20
0
23 25 27 29 31 33 35 37 39 41 43 45 47 49 51 53 55 57 59
Age
Shocks
No shocks

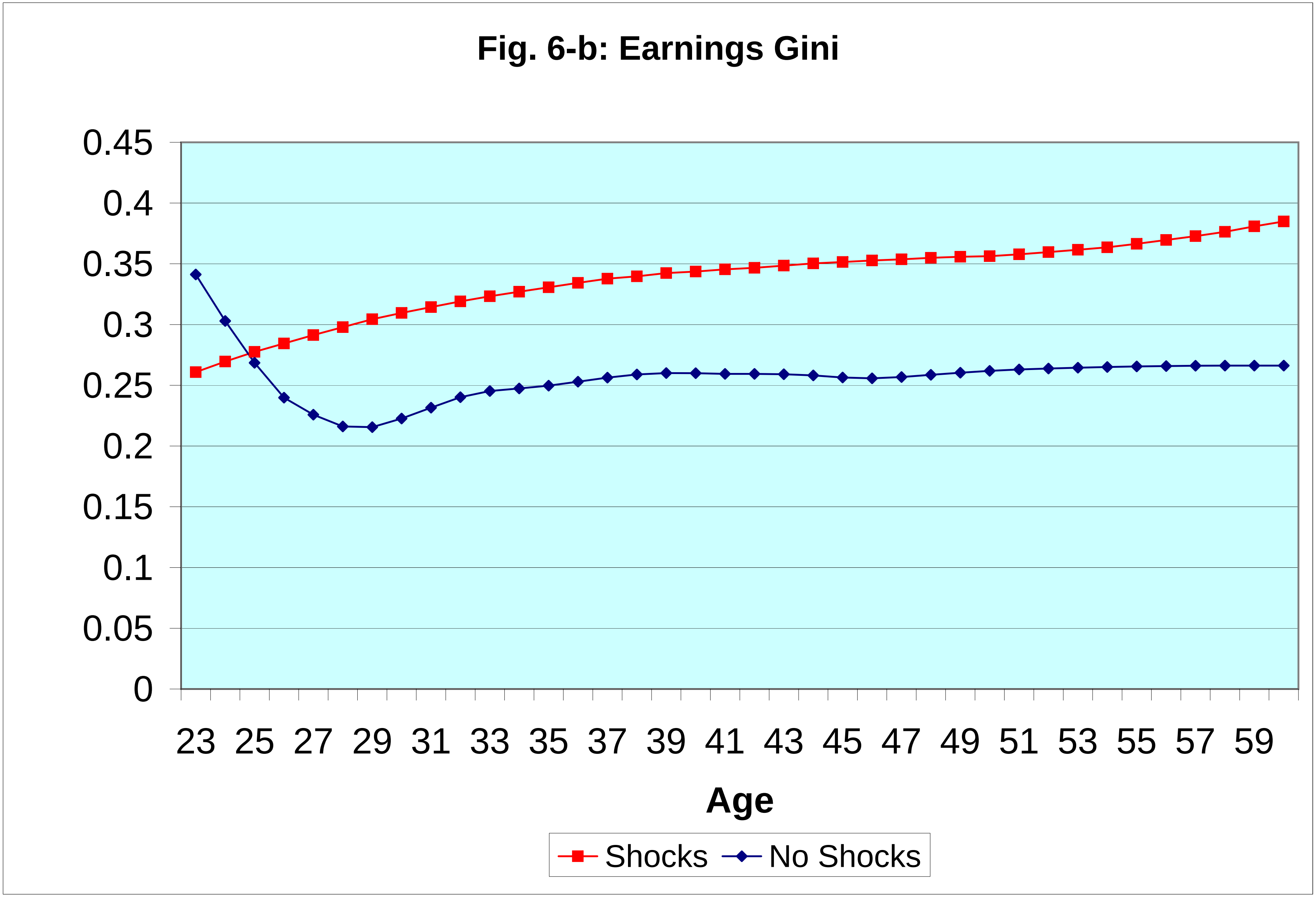


Fig. 6-b: Earnings Gini
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
23 25 27 29 31 33 35 37 39 41 43 45 47 49 51 53 55 57 59
Age
Shocks
No Shocks