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Competitive Equilibria With Limited Enforcement*

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ABSTRACT

We show how to decentralize constrained efficient allocations that arise from enforcement constraints between sovereign nations. In a pure exchange economy these allocations can be decentralized with private agents acting competitively and taking as given government default decisions on foreign debt. In an economy with capital these allocations can be decentralized if the government can tax capital income as well as default on foreign debt. The tax on capital income is needed to make private agents internalize a subtle externality. The decisions of the government can arise as an equilibrium of a dynamic game between governments.

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In this paper we study equilibria in economies in which there is limited ability to enforce credit arrangements between sovereign nations. In previous work (Kehoe and Perri [12]) we have discussed how this limited ability manifests itself in *enforcement constraints* which require that in each period and state, allocations can be enforced only if their value is greater than it would be if the country were excluded from all further intertemporal and interstate trade. This friction captures in a simple way the difficulties of enforcing contracts between sovereign nations that involve large transfers of resources backed only by promises to repay. This type of friction turns out to be useful to explain the international macroeconomic comovements.

Our recent work focuses on planning problems with enforcement constraints, or *constrained efficient allocations,* but does not analyze in detail how these allocations can be decentralized. Here we do that detailed analysis. We follow the literature on sovereign debt in assuming that the decision to partially or completely default on foreign debt is made by the government of the borrowing country, the *domestic government.* We abstract completely from any incentive of private agents to default.

The assumptions of the sovereign debt literature are motivated by historical experience. Foreign creditors cannot easily use the domestic legal system to pursue legal claims against borrowers who do not repay their loans. Therefore, international loans typically involve the domestic government. Either the loans are made directly to the government, which then relends the funds through a domestic intermediary, or the loans are nominally made directly to private entities, like firms, banks, and private households, but the government guarantees the collection of debt and the repayment to foreigners. Either way, it is the government and not a private agent that decides to default on loans to foreigners. Of course, private agents can decide to default on their obligations to the government just as they can default on their obligations to other domestic agents. These private defaults are subject to the domestic legal system, and we abstract from them, as does most of the sovereign debt literature, in order to focus the attention on the international elements. (See the survey by Eaton and Fernandez [7] for a further discussion of the empirical motivation for these assumptions[1].)

In our economy, we model partial or complete default on foreign debt as a decision of the domestic government. That government has the ability to force its domestic agents to repay the government for the foreign loans that it channeled to them, but then the government can choose how much of these collected funds to pass on to foreigners. For simplicity, we assume that if the government decides to partially or completely default on debt to foreigners, it redistributes the unpaid portion of these funds to the domestic agents in a lump-sum fashion. Thus, domestic agents always have to fully repay the government for the loan. The default decision by the government affects the problems of domestic agents by affecting the interest rates at which foreigners will lend to them.

We begin with a pure exchange economy with two countries and a large number of identical consumers in each. We set up a planning problem with enforcement constraints and show how the resulting constrained efficient allocations can be characterized by a transition law for the ratio of marginal utilities of consumers across countries together with a resource constraint. We show that the constrained efficient allocations can be decentralized as a competitive equilibrium in which private agents take as given the default decisions of governments.

We then define a dynamic game in which the governments of the countries optimally choose the default rates as part of the equilibria, while private agents act competitively, taking the government default decisions as exogenous. We show that any constrained efficient allocation can be supported as an equilibrium of this dynamic game. We do so by showing that any allocation that satisfies the resource constraints and the enforcement constraints can be supported as an equilibrium in the dynamic game. The constrained efficient allocation thus has the interpretation as the best such allocation. In this sense, our economy is a standard competitive environment in which limited international risk-sharing arises endogenously from the limited enforcement of international contracts and the strategic interactions between governments.

We then add capital to the model, so that the economy is a two-country standard growth model with enforcement constraints. Here the constrained efficient allocations cannot be decentralized when the only instrument available to the government is the default rate on foreign loans. This is because in the planning problem with binding enforcement constraints, the Euler equation for capital accumulation is necessarily distorted away from the first-best.

The distortions in the Euler equation arise from two effects. One is that the planner has a different intertemporal marginal rate of substitution in consumption than would prevail without the enforcement constraints. In particular, a decrease in consumption in one period that is followed by an increase in consumption in the next ends up relaxing one more enforcement constraint than it tightens. This leads to an extra benefit for higher investment beyond the usual one.

The other effect behind the distortions in the Euler equation is that a larger amount of capital makes the enforcement constraint tighter by raising the value of financial autarky. Together these effects introduce extra terms in the Euler equation that break the link between the intertemporal marginal rates of substitution and transformation. In the decentralized equilibrium, private agents equate these marginal rates, and the constrained efficient allocations cannot be decentralized with just government default on loans. The intuition is that private agents do not internalize the indirect effects their actions have on the decisions of the government.

We then augment the government's instruments by also allowing it to tax capital income. With both such instruments—debt default and capital income taxes—the constrained efficient allocations can be decentralized. When the capital income tax is set appropriately, it both aligns the intertemporal marginal rates of substitution of the private agents with those of the planner and makes private agents internalize the external effect generated by investment. It is then easy to show that any constrained efficient allocation can be supported as the equilibrium of a dynamic game in which governments choose both default rates on foreign debt and domestic capital income tax rates.

One implication of this decentralization is that it provides a rationale for capital income taxes. This is in contrast to the optimal tax literature in which optimal capital income taxes are often zero. (For a survey of this literature, see Chari and Kehoe [6].)

The main contribution of this work is to show how limited international risk-sharing can endogenously arise in the equilibrium of an appropriately defined game with competitive private agents. As such, this work builds on both the literature on international debt—such as the studies of Eaton and Gersovitz [8], Kletzer and Wright [13], and Manuelli [17] and those surveyed by Eaton and Fernandez [7]—and the literature on debt-constrained asset markets, particularly the studies of Alvarez and Jermann [2], Attanasio and Ríos-Rull [3], Kehoe and Levine [10, 11], Kocherlakota [14], and Ligon, Thomas, and Worrall [15].

Consider first the international debt literature. In most of this literature private competitive agents in the borrowing country are not explicitly modeled; instead, a game is set up between a large agent, often thought of as the government of the borrowing country and some foreign lenders. In some of this literature it is argued that efficient outcomes can be achieved only if either lenders ration credit or lenders impose seniority clauses. (See the survey by Eaton and Fernandez [7].)

In the debt-constrained asset market literature, private agents are explicitly modeled as competitive, but the constraints that private consumers face are not explicitly chosen by any agent as part of the equilibrium. For example, in the work of Kehoe and Levine [10], the enforcement constraints are built directly into the commodity space. Alvarez and Jermann [2] go the farthest and show how appropriately set constraints on debt can decentralize the constrained efficient allocations as a competitive equilibrium. Even in that work, however, the debt constraints are not chosen by any agent. Alvarez and Jermann [2] show, rather, that if the debt constraints are appropriately set, then the allocations of interest can be decentralized.

In some interesting work Jeske [9] and Wright [21] also analyze competitive equilibria in pure exchange economies with limited enforcement. Jeske compares economies in which private agents are allowed to borrow internationally and make default decisions with economies in which the international borrowing and default is done solely by the government. In contrast, in our setup we allow private agents to borrow but the government makes the default decisions. Wright [21] also considers constrained efficient allocations in an economy with limited enforcement but he mainly focuses on a decentralization in which the decision to repudiate the debt is made by private agents and not by governments. In Wright's work taxes on international borrowing are used by the government to prevent private agents from borrowing too much. In our setup anticipations of the government's default decisions raises the price of foreign debt to a level such that taking as given this price, the private agents optimally choose the correct amount.

Our work goes beyond the literatures on international debt and debt-constrained asset markets. In contrast to the international debt literature we explicitly model the behavior of private agents in both countries. Interestingly, in contrast to that literature we find the economy achieves the relevant efficient outcome with no need for credit-rationing or seniority clauses. In contrast to the debt-constrained asset market literature, the limited risk sharing arises from more primitive decisions taken by agents that are explicitly modelled, namely the governments. In particular, the decisions to default by the government, which are the mechanism through which international risk-sharing is limited, are derived endogenously as equilibria of a dynamic game between governments with competitive private agents.

Moreover, in contrast, to almost all of the literature we consider an economy with capital. (See Seppala [19] and Wright [20] for exceptions.) With capital the governments need a second instrument in order to decentralize the constrained efficient allocations. We show that an appropriately set capital income tax together with the default rates will support the decentralize the constrained efficient allocations.

1. CONSTRAINED EFFICIENT ALLOCATIONS

Consider the following deterministic pure exchange economy, which is a special case of the stochastic pure exchange economy studied by Alvarez and Jermann [2] and the stochastic production economy we have studied (Kehoe and Perri [12]). We will show here that constrained efficient allocations in this economy can be decentralized when private agents take as given some exogenously set default decisions by governments.

1.1. The World Economy

Our theoretical world economy consists of two countries, $i = 1, 2$, each represented by a large number of identical, infinitely lived consumers and a time-varying deterministic endowment of a single homogeneous consumption good. The endowment of country i in time period t is y_{it} while consumers in country i have utility, or *preferences,* of the form $\sum_{t=0}^{\infty} \beta^t U(c_{it})$, where c_{it} denotes consumption of the endowment good by consumers in country i in t and β denotes the discount factor. The *resource constraints* are given by

$$c_{1t} + c_{2t} = y_{1t} + y_{2t}. \tag{1}$$

We assume that for country $i = 1, 2$, all endowments $y_{it} \in [\underline{y}, \bar{y}]$ for some finite, strictly positive constants $\underline{y}$ and $\bar{y}$.

The presence of limited enforcement of international contracts implies that in each period each country has the option of reneging on any outstanding obligations and living in *autarky* forever after. This possibility imposes on any equilibrium allocation a set of *enforcement constraints* which require that at every point in time, each country prefers the equilibrium allocation over the allocation it could get if it were in autarky from then on. These enforcement constraints are of the form

$$\sum_{s=t}^{\infty} \beta^{s-t} U(c_{is}) \geq V_{it} = \sum_{s=t}^{\infty} \beta^{s-t} U(y_{is}) \tag{2}$$

where V_{it} denotes the value of autarky for country i from period t on, which is given by the value of utility in which consumers simply consume their endowment from t on.

The constrained efficient allocations of this economy solve the planning problem of maximizing a weighted sum of the discounted utilities:

$$\max \left[\lambda_1 \sum_{t=0}^{\infty} \beta^t U(c_{1t}) + \lambda_2 \sum_{t=0}^{\infty} \beta^t U(c_{2t}) \right] \tag{3}$$

subject to the resource constraints (1) and the enforcement constraints (2) for country $i = 1, 2$ and all periods t, where λ_1 and λ_2 are nonnegative initial weights on the two countries' utilities.

An allocation $\{c_{1t}, c_{2t}\}_{t=0}^{\infty}$ is *constrained efficient* if it solves the planning problem for some nonnegative planning weights λ_1 and λ_2. We characterize these allocations as follows. Let $\beta^t \mu_{it}$ and γ_t denote the multipliers on the enforcement constraints and the resource constraints, respectively. The first-order conditions are, then

$$\beta^t U'(c_{it})[\lambda_i + \mu_{i0} + \mu_{i1} + \ldots + \mu_{it}] = \gamma_t. \tag{4}$$

From (4), we see that an increase in consumption c_{it} in period t has two effects. It has the standard effect of raising the objective function by $\beta^t U'(c_{it})\lambda_i$. It also relaxes all of the incentive constraints from period 0 through period t. This effect has value $\beta^t U'(c_{it})(\mu_{i0} + \ldots + \mu_{it})$. Using

(4), we see that the planner's effective intertemporal marginal rate of substitution between goods in period t and goods in period $t+1$ is

$$\frac{\gamma_{t+1}}{\gamma_t} = \frac{\beta U'(c_{it+1})}{U'(c_{it})} + \frac{\beta U'(c_{it+1})}{U'(c_{it})} \frac{\mu_{it+1}}{M_{it}} \tag{5}$$

where $M_{it} = \lambda_{i0} + \mu_{i0} + \ldots + \mu_{it}$. The first term on the right side of (5) is the standard one that arises from changing the value of the objective function by moving one unit of consumption from period t to period $t+1$. The second term captures the following effect on incentives. Decreasing consumption in period t tightens the incentive constraints from period 0 to period t, while increasing consumption in period $t+1$ relaxes these constraints from period 0 to period $t+1$. The net effect on the incentive constraints by such a change is to relax the constraint in period $t+1$. The second term on the right side of (5) is nonnegative, and thus, the planner has a higher intertemporal marginal rate of substitution than the standard one. From (5) it follows that

$$\frac{\beta U'(c_{1t+1})}{U'(c_{1t})} \left(1 + \frac{\mu_{1t+1}}{M_{1t}}\right) = \frac{\beta U'(c_{2t+1})}{U'(c_{2t})} \left(1 + \frac{\mu_{2t+1}}{M_{2t}}\right). \tag{6}$$

It should be clear that in a given period $t+1$, both incentive constraints cannot bind. Thus, there are three binding patterns: $\mu_{1t+1} > 0$ and $\mu_{2t+1} = 0$ or $\mu_{1t+1} = 0$ and $\mu_{2t+1} > 0$ or $\mu_{1t+1} = \mu_{2t+1} = 0$. For example, if $\mu_{1t+1} > 0$ and $\mu_{2t+1} = 0$, then (6) implies that shifting consumption in period $t+1$ from consumer 2 to consumer 1 has the additional benefit of relaxing the incentive constraint for consumer 1 in period $t+1$ over and above the standard effects on marginal utility.

It is convenient to have notation for the consumer with the higher intertemporal marginal rate of substitution and the consumer with the lower rate. Let

$$q_{t,t+1} = \max_i \frac{\beta U'(c_{it+1})}{U'(c_{it})} \tag{7}$$

$$p_{t,t+1} = \min_i \frac{\beta U'(c_{it+1})}{U'(c_{it})}. \tag{8}$$

In our decentralization, $q_{t,t+1}$ and $p_{t,t+1}$ correspond to the marginal rate of substitution for the lender and the borrower, respectively. We then have a version of a lemma established by Alvarez and Jermann [2]:

Lemma. Let $\{c_{1t}, c_{2t}\}$ be a constrained efficient allocation. If the enforcement constraint for consumer j in period $t+1$ is slack, then

$$\frac{\beta U'(c_{jt+1})}{U'(c_{jt})} = q_{t,t+1}. \tag{9}$$

If the enforcement constraint for consumer j in period $t+1$ binds, then

$$\frac{\beta U'(c_{jt+1})}{U'(c_{jt})} = p_{t,t+1}. \tag{10}$$

Proof. If the enforcement constraint for consumer 1 is slack at $t+1$, then $\mu_{1t+1} = 0$. Eq. (6) and the fact that $\mu_{2t+1} \geq 0$ and $M_{2t} > 0$ imply that the marginal rate of substitution of consumer 1 is higher than that of consumer 2, so (9) holds. If the enforcement constraint for consumer 1 binds at $t+1$, then $\mu_{1t+1} > 0$ and $\mu_{2t+1} = 0$. Eq. (6) plus the fact that $M_{1t} > 0$ imply that the marginal rate of substitution of consumer 1 is lower than that of consumer 2, and (10) holds. ■

We will be most interested in allocations for which the present value of the allocation, at the appropriately defined intertemporal prices, is finite for each consumer. Letting $q_{0,t} = q_{0,1}q_{1,2}\ldots q_{t-1,t}$, we say that an allocation $\{c_{1t}, c_{2t}\}_{t=0}^{\infty}$ has *high implied interest rates* if for $i = 1, 2$,

$$\sum_{t=0}^{\infty} q_{0,t}(y_{1t} + y_{2t}) < \infty. \tag{11}$$

Here $q_{t,t+1}$ is the marginal rate of substitution for whichever country's representative consumer is unconstrained in period $t+1$. Typically, in some periods, one country's consumer will be unconstrained while in other periods, the other country's consumer will be unconstrained. Thus, the product of these marginal rates $q_{0,t}$ does not represent any single consumer's marginal rate of substitution between periods 0 and t, but rather is a mixture of both representative consumers' marginal rates. The high implied interest rate condition guarantees that in the decentralized equilibrium the present value of endowments are finite. We use it to show that constructed assets are finite and that the consumer's transversality condition holds.

1.2. Decentralization With Government Default

Now we discuss how to decentralize the constrained efficient allocations as a competitive equilibrium with government default decisions taken as given. We show that if these default decisions are appropriately chosen, then the constrained efficient allocations can be decentralized. (In the next section, we will allow the governments to purposefully choose these default decisions.)

In this economy, the government of each country collects any repayments its consumers make on foreign loans and then decides how much of these repayments to pass on to foreigners. The idea is that all loans from foreigners to domestic consumers are channeled through the domestic government. For simplicity, we assume that the government rebates in a lump-sum fashion any net revenues it takes in. Except for these government policies, private markets function perfectly.

We begin by setting up a competitive equilibrium with government default. Consider the consumer problem and the government budget constraint for some arbitrarily given sequence of government policies and prices. Throughout we will focus on country 1; the notation for country 2 is analogous. It is convenient to define separate variables for saving and for borrowing. We let $s_{1t+1} \geq 0$ denote the savings, or assets, of a consumer in country 1, $b_{1t+1} \geq 0$ denote that consumer's borrowings, or liabilities, and $\tau_{1t} \in [0, 1]$ denote the default rate by the government of country 1 on foreign lenders, which here are country 2 consumers.

The problem for a consumer in country 1 is to maximize utility

$$\sum_{t=0}^{\infty} \beta^t U(c_{1t})$$

subject to the budget constraint

$$c_{1t} + P_{t,t+1}(s_{1t+1} - b_{1t+1}) = y_{1t} + (1 - \tau_{2t})s_{1t} - b_{1t} + T_{1t}; \tag{12}$$

the nonnegativity constraints $s_{it+1}, b_{it+1} \geq 0$; and bounds on debt $b_{1t+1} \leq \bar{b}$, where $\bar{b}$ is a large positive constant. Here $P_{t,t+1}$ is the price of a consumption good in $t+1$ in period t units, τ_{2t} is the default rate chosen by country 2's government of payments s_{1t} that country 2 consumers make to country 1 consumers, and T_{1t} is the lump-sum rebates by the government of country 1 to its consumers. The initial assets s_{i0} and liabilities b_{i0} are given.

The government budget constraint in country 1 is $T_{1t} = \tau_{1t} b_{1t}$, so that any revenues taken in by the government through partial or complete default is rebated to consumers.

A *competitive equilibrium with default rates* $\{\tau_{1t}, \tau_{2t}\}_{t=0}^{\infty}$ together with initial assets and liabilities $\{s_{i0}, b_{i0}\}_{i=1,2}$ consists of an allocation $\{c_{1t}, c_{2t}\}_{t=0}^{\infty}$, assets $\{s_{1t+1}, s_{2t+1}\}_{t=0}^{\infty}$, liabilities $\{b_{1t+1}, b_{2t+1}\}_{t=0}^{\infty}$, and prices $\{P_{t,t+1}\}_{t=0}^{\infty}$ such that $\{c_{it}, s_{it+1}, b_{it+1}\}$ solves the consumer problem for each i, and markets clear, so that $s_{1t+1} = b_{2t+1}$ and $b_{1t+1} = s_{2t+1}$ and the resource constraints (1) hold.

To understand the budget constraints of the consumer and the government, suppose that in period $t-1$, a consumer in country 1 lends $P_{t-1,t} s_{1t}$ in exchange for a promise to receive, in period t, the amount s_{1t} minus the portion withheld by the government, namely, $\tau_{2t} s_{1t}$. Consumers in country 2 repay the full amount owed, $s_{1t} = b_{2t}$, but the government of country 2 repays to country 1 consumers only part of that, $(1 - \tau_{2t}) s_{1t}$, and redistributes the rest to its consumers in a lump-fashion.

For brevity, from now on we let U'_{it} denote $U'(c_{it})$. With this notation, the first-order conditions for consumer 1's problem are

$$P_{t,t+1} U'_{1t} \geq \beta U'_{1t+1} (1 - \tau_{2t+1}) \tag{13}$$

with equality if $s_{1t+1} > 0$, so that country 1 is lending to country 2; and

$$P_{t,t+1} U'_{1t} \leq \beta U'_{1t+1} \tag{14}$$

with equality if $b_{1t+1} > 0$, so that country 2 is lending to country 1. Here and throughout we assume that the debt constraint $b_{1t+1} \leq \bar{b}$ does not bind. The transversality condition is

$$\lim_{t\to\infty} \beta^t P_{t,t+1} U'_{1t}(s_{1t+1} - b_{1t+1}) = 0. \tag{15}$$

We have the following proposition.

Proposition 1. Any allocation that satisfies the resource constraints (1) and the enforcement constraints (2) and has high implied interest rates (11) can be decentralized as a competitive equilibrium with government default for some appropriate choice of initial assets.

Proof. We decentralize the given allocation as follows. We first set the intertemporal prices $P_{t,t+1} = p_{t,t+1} = \min_i[\beta U'(c_{it+1})/U'(c_{it})]$. These prices will correspond to the borrower's marginal rate of substitution. We set the default rates as follows:

$$\tau_{1t+1} = \left\{ \begin{array}{c} 1 - p_{t,t+1}/q_{t,t+1} \text{ if } U'_{1t+1}/U'_{1t} > U'_{2t+1}/U'_{2t} \\ 0 \text{ otherwise} \end{array} \right\} \tag{16}$$

$$\tau_{2t+1} = \left\{ \begin{array}{c} 0 \text{ if } U'_{1t+1}/U'_{1t} > U'_{2t+1}/U'_{2t} \\ 1 - p_{t,t+1}/q_{t,t+1} \text{ otherwise} \end{array} \right\} \tag{17}$$

for $t \geq 0$ and $\tau_{10} = 0$, $\tau_{20} = 0$. Note that if $U'_{1t+1}/U'_{1t} > U'_{2t+1}/U'_{2t}$, then country 1 is lending to country 2. The constructed default rates lie between 0 and 1 and satisfy

$$(1 - \tau_{1t+1})(1 - \tau_{2t+1}) = \frac{p_{t,t+1}}{q_{t,t+1}}. \tag{18}$$

For assets and liabilities, we set

$$s_{1t+1} - b_{1t+1} = \frac{q_{t,t+1}}{p_{t,t+1}} \sum_{s=t+1}^{\infty} q_{t+1,s}(c_{1s} - y_{1s}) \tag{19}$$

for $t \geq 0$, and for initial assets, we set

$$s_{10} - b_{10} = \sum_{s=0}^{\infty} q_{0,s}(c_{1s} - y_{1s}) \tag{20}$$

where we know that the right sides of (19) and (20) are finite because of the high interest rate condition (11). Here $q_{t,t+1}$ corresponds to the marginal rate of substitution for the lenders. If the right side of (19) is nonnegative, we set $b_{1t+1} = 0$; if the right side of (19) is negative, we set $s_{1t+1} = 0$. We set s_{10} and b_{10} analogously from (20). Eq. (19) defines the assets and liabilities chosen by consumers in equilibrium while (20) defines the initial assets and liabilities that are exogenously given to consumers.

We can see that the constructed prices, repayment rates, and assets and liabilities are a competitive equilibrium as follows. To check the constructed prices, notice that in equilibrium in any period t, there are three possibilities. One is that $U'_{1t+1}/U'_{1t} > U'_{2t+1}/U'_{2t}$. Here country 1 is lending to country 2, $s_{1t+1} = b_{2t+1} > 0$, $s_{2t+1} = b_{1t+1} = 0$, and

$$p_{t,t+1} = \beta \frac{U'_{1t+1}}{U'_{1t}}(1 - \tau_{2t+1}) = \beta \frac{U'_{2t+1}}{U'_{2t}}. \tag{21}$$

Another possibility is that $U'_{1t+1}/U'_{1t} < U'_{2t+1}/U'_{2t}$. Here country 2 is lending to country 1, so that $s_{2t+1} = b_{1t+1} > 0$, $s_{1t+1} = b_{2t+1} = 0$, and

$$p_{t,t+1} = \beta \frac{U'_{1t+1}}{U'_{1t}} = \beta \frac{U'_{2t+1}}{U'_{2t}}(1 - \tau_{1t+1}). \tag{22}$$

A third possibility is that $U'_{1t+1}/U'_{1t} = U'_{2t+1}/U'_{2t}$ and

$$p_{t,t+1} = \beta \frac{U'_{1t+1}}{U'_{1t}} = \beta \frac{U'_{2t+1}}{U'_{2t}}. \tag{23}$$

From (21)–(23) hold, it is clear that the price $p_{t,t+1}$ satisfies (8), which, recall, is the marginal rate of substitution for the borrower. To check the constructed default rates, note the following. If (21) holds, then $U'_{1t+1}/U'_{1t} = q_{t,t+1}$; if (22) holds, then $U'_{2t+1}/U'_{2t} = q_{t,t+1}$; if (23) holds, then

$p_{t,t+1} = q_{t,t+1}$. Clearly, then, the constructed default rates (16) and (17) satisfy (21)–(23).

To check that the constructed assets and liabilities are budget feasible, consider an arbitrary period t and substitute (19) and the budget constraint of the government $T_{1t} = \tau_{1t} b_{1t}$ into the budget constraint of the consumer (12). Then rearrange to obtain

$$q_{t,t+1} \sum_{s=t+1}^{\infty} q_{t+1,s}(c_{1s} - y_{1s}) = y_{1t} - c_{1t} + (1 - \tau_{2t})s_{1t} - b_{1t}(1 - \tau_{1t}). \tag{24}$$

Again, there are three possibilities. One is that $U'_{1t}/U'_{1t-1} > U'_{2t}/U'_{2t-1}$. In period $t-1$, country 1 is lending to country 2, $s_{1t} > 0$, and $b_{1t} = 0$. In this case, $1 - \tau_{2t} = p_{t-1,t}/q_{t-1,t}$. Substituting into (24) and rearranging, we get

$$s_{1t} - b_{1t} = \frac{q_{t-1,t}}{p_{t-1,t}} \sum_{s=t}^{\infty} q_{t,s}(c_{is} - y_{is}) \tag{25}$$

which is the same as (19), shifted back one period. In the other two possibilities ($U'_{1t}/U'_{1t-1} < U'_{2t}/U'_{2t-1}$ and $U'_{1t}/U'_{1t-1} = U'_{2t}/U'_{2t-1}$), the same logic applies, and we obtain the same expression as in (25) for assets in period t. Since this logic applies for every period, we have shown that the definition of assets is consistent with the given allocation and with the budget constraint of the consumer.

The final step in the proof of Proposition 1 is to show that at the constructed allocations, if the high implied interest rate condition (11) holds, then the transversality condition (15) holds. We need only show that $\lim_{t\to\infty} \beta^t p_{t,t+1} U'_{1t}(b_{1t+1} - s_{1t+1}) = 0$. Now using (19) and the relation $q_{t,s} = q_{t,t+1}q_{t+1,s}$, we can write

$$\begin{aligned} &\lim_{t\to\infty} \beta^t p_{t,t+1}(b_{1t+1} - s_{1t+1}) \\ &= \lim_{t\to\infty} \beta^t U'_{1t} \sum_{s=t+1}^{\infty} q_{t,s}(y_{1s} - c_{1s}). \end{aligned} \tag{26}$$

Using the result that $\beta^t U'_{1t}/U'_{10} \leq q_{0,t}$, so that $\beta^t U'_{1t} q_{t,s} = U'_{10}(\beta^t U'_{1t}/U'_{10})q_{t,s} \leq U'_{10} q_{0,t} q_{t,s} = U'_{10} q_{0,s}$, we know that (26) is less than or equal to

$$U'_{10} \lim_{t\to\infty} \sum_{s=t+1}^{\infty} q_{0,s}(y_{1s} - c_{1s}) \leq U'_{10} \lim_{t\to\infty} \sum_{s=t+1}^{\infty} q_{0,s}(y_{1s} + y_{2s}) = 0 \tag{27}$$

where the equality in (27) follows from (11). ■

The basic idea of the construction is as follows. From the consumer budget constraint (12), it is clear that the price $p_{t,t+1}$ has the following interpretation. For a borrower, it is the amount a borrower receives in period t for a payment of one unit in $t+1$. For a lender, it is the amount the lender gives in t for a payment of one unit in $t+1$ minus whatever the percentage amount that the government of the borrowing country decides to default. Clearly, in the decentralized equilibrium, this price will be equal to the borrower's marginal rate of substitution. From (21) and (22), it is clear that the marginal rate of substitution of the borrower equals that of the lender once this lender's rate has been reduced by the default rate. Hence, we can use the ratio of these marginal rates to construct the implicit default rates. Finally, when calculating the relevant present discounted values for assets and liabilities in (19), the relevant price is the borrower's marginal rate of substitution.

2. ENDOGENIZING THE DEFAULT DECISIONS

In our decentralization, we have used allocations that satisfy enforcement constraints, resource constraints, and the high interest rate condition to construct the appropriate default rates that decentralize the given allocations, but we have not offered a story about where these default rates come from. Here we provide a story for how the constructed default rates may come out of an equilibrium of a dynamic game with both government behavior and consumer behavior endogenous.

2.1. The Dynamic Game

We set up this dynamic game as follows. In each period, the governments and the consumers can vary their decisions, depending on the history of government policies up to the time the decision is made. We let $\pi_t = (\tau_{1t}, \tau_{2t})$ denote the two governments' policies in period t. At the beginning of period t, the government of each country chooses a current policy as a function of the history of past government policies $h_{t-1} = (\pi_0, ..., \pi_{t-1})$ together with a contingency plan for setting future policies for all possible future histories. Let $\tau_{it}(h_{t-1})$ denote the period t default rate (so that $1 - \tau_{it}(h_{t-1})$ is the repayment rate) chosen by the government of country i when faced with history h_{t-1}. After the government sets the current policies, consumers make their decisions. Faced with the history $h_t = (h_{t-1}, \pi_t)$, consumers in country i choose their period t consumption, assets, and liabilities, denoted $f_{it}(h_t) = \big(c_{it}(h_t),\, s_{it+1}(h_{it}), b_{it+1}(h_t)\big)$. The prices are a function of the government policy history and are denoted $p_{t,t+1}(h_t)$. Let $\tau = (\tau_1, \tau_2)$, and let τ_i denote the infinite sequence of functions (τ_{it}). Use similar notation for the other variables.

For some given initial assets and liabilities, a *sustainable equilibrium* is a triple (τ, f, p) such that three conditions are satisfied:

(i) For $i = 1, 2$, for every history of government policies h_t, the consumer allocations $f_{is}(h_s)$ for $s = t, ...,$ solve

$$\max \sum_{s=t}^{\infty} \beta^{s-t} U(c_{is})$$

subject to

$$c_{1s} + p_{s,s+1}(h_s)(s_{1s+1} - b_{1s+1})$$

$$= y_{1s} + [1 - \tau_{2s}(h_{s-1}) s_{1s}(h_{s-1})] - b_{1s}(h_{s-1}) + T_{1s}(h_{s-1})$$

where the future histories' policies and prices are induced from h_t, τ, and p in the obvious way. That is, $h_{t+1} = \big(h_t, \tau_{t+1}(h_t)\big)$ and $h_{t+2} = \Big(h_t,\tau_{t+1}(h_t),\tau_{t+2}\big(h_t, \tau_{t+1}(h_t)\big)\Big)$, and given these induced future histories, the policies and prices are given by $\tau_s(h_{s-1})$ and $p_s(h_s)$.

(ii) For every history h_t, markets clear and the government budget constraint holds for $s = t, ...,$ so that $c_{1s}(h_s) + c_{2s}(h_s) = y_{1s} + y_{2s}$ as well as $s_{1s+1}(h_s) = b_{2s+1}(h_s)$, $s_{2s+1}(h_s) = b_{1s+1}(h_s)$, and $T_{1s}(h_{s-1}) \equiv \tau_{1s}(h_{s-1}) b_{1s}(h_{s-1})$, where the future histories h_s are induced from τ in the obvious way.

(iii) For every history h_{t-1}, country 1's government policies from t on, τ_{1s} for all $s \geq t$, solve

$$\max \sum_{s=t}^{\infty} \beta^{s-t} U\big(c_{1s}(h'_{s-1})\big),$$

where $h_t' = \big(h_{t-1},(\tau_{1t}'(h_{t-1}), \tau_{2t}'(h_{t-1}))\big)$ and $h_{t+1}' = \big(h_t,(\tau_{1t+1}'(h_t), \tau_{2t+1}'(h_t))\big)$ and so on. A similar condition holds for the government of country 2.

Notice that in this definition of a sustainable equilibrium, we require that both the governments and the consumers act optimally for every history of policies—even for histories not induced by the governments' policy plans. This requirement is analogous to the requirement of perfection in a game. In this definition, the consumers act competitively in that they take current policies and prices and the evolution of future histories as unaffected by their actions. The governments are not competitive. The government of country 1, for example, takes the allocation rules f_1 and f_2, the price function p, and the policy plan of the government of country 2, τ_2, as given. But the government of country 1 realizes that it can affect outcomes both directly, by changing the default rate on its foreign loans, and indirectly, by affecting the evolution of the future history and thus affecting the policies chosen by the other government, the allocations chosen by the consumers, and the prices.

2.2. Outcomes of a Sustainable Equilibrium

Recall that a sustainable equilibrium (τ, f, p) is a sequence of functions that specify policies, allocations, and prices for all possible government policy histories. Thus, when we start from the null history in period 0, a sustainable equilibrium induces a particular sequence of policies, allocations, and prices that we denote by (π, x, p). We call this the *outcome* induced by the sustainable equilibrium. In what follows, we adapt the work of Chari and Kehoe [4, 5], which builds on the work of Abreu [1], to characterize this outcome.

We first construct a sustainable equilibrium that we call the *autarky equilibrium.* We then characterize the allocations that can be induced by reverting to this autarky equilibrium after deviations. We define the autarky policy plans τ^a, allocation rules f^a, and price rules p^a starting from some given initial assets and liabilities as follows. The policy plan specifies complete default, namely, $\tau_{it}^a(h_{t-1}) = 1$, for all i and t. Given any history h_t, the autarky allocations $\big(c_{it}^a(h_t), s_{it+1}^a(h_t), b_{it+1}^a(h_t)\big)$ are given by $c_{it}^a(h_t) = y_{it}$, while the autarky prices of debt and the quantities of assets and liabilities are identically zero, so $p_{t,t+1}^a(h_t) = s_{it+1}^a(h_t) = b_{it+1}^a(h_t) = 0$. The utility of autarky for consumer i in period t is V_{it}.

We now characterize the outcomes that can be sustained by a set of plans called the *revert-to-autarky* plans, which are defined as follows. For an arbitrary sequence of policies, allocations, and prices (π, x, p), these plans specify continuation with the candidate sequences (π, x, p) as long as the specified policies have been chosen in the past; otherwise, the plans specify the revert-to-autarky plans (τ^a, f^a, p^a). We then have

Proposition 2. An arbitrary triple of sequences (π, x, p) that satisfies the high implied interest rate condition (11) can be sustained by the revert-to-autarky plans if and only if the triple is a competitive equilibrium with government default for some choice of initial assets and if, for $i = 1, 2$ for every t, the following inequality holds:

$$\sum_{s=t}^{\infty} \beta^{s-t} U(c_{is}) \geq V_{it}. \tag{28}$$

Proof. Suppose, first, that the sequences of policies, allocations, and prices (π, x, p) can be sustained by the revert-to-autarky plans; that is, suppose the associated revert-to-autarky plans (τ, f, p) constitute a sustainable equilibrium. From the definition of a sustainable equilibrium, consumer optimality requires that x maximize consumer welfare in period 0. This requirement together with market-clearing ensures that this sequence is a competitive equilibrium in period 0.

Next, we claim that inequality (28) holds for all i and t. Note that a feasible policy for the government of i in t is to choose the autarky policies for all $s \geq t$ by taxing repayments to consumers in the other country at rate 1. This policy will lead to a continuation utility of V_{it}^a, and hence, optimality of government policy ensures that (28) holds.

Now suppose that some arbitrary triple of sequences (π, x, p) satisfies the proposition's conditions. We show that the associated revert-to-autarky plans constitute a sustainable equilibrium. Consider, first, histories for which there have been no deviations from π before t. Since (π, x, p) is a competitive equilibrium in period 0, x is optimal for consumers in period 0 given π and p, and thus, the continuation of x is optimal for consumers when they are faced with the continuation of π and p. In terms of government optimality, consider the situation of the government of country

1. If it deviates in period t, then the consumers in both countries and the government of country 2 will revert to the autarky policy plans and the autarky allocation rules from period t on. Under these allocation rules, country 2 consumers will never lend to country 1 consumers, regardless of the policies chosen by the government of country 1. Thus, the best the government of country 1 can obtain is the value of autarky from then on given by the right side of (28). Given the assumed inequality, then, sticking to the specified plan is optimal.

Consider, next, histories with a deviation from π before t. Clearly, the autarky plans from then on are sustainable. From a consumer's point of view, since no debt will be repaid, lending is not optimal. The price of debt is zero since the value to a potential lender in the other country of a promise to pay one unit tomorrow, net of taxes equal to one unit, is worthless. Thus, the consumer is indifferent among all amounts to borrow or lend because all have value 0 and all pay 0. From a government's point of view, given that the other government never allows its consumers to repay their debts outside the country, regardless of the first government's actions, it is optimal for the first government to prevent its own consumers from repaying their debts outside the country. ■

Combining Propositions 1 and 2, we immediately obtain the following proposition:

Proposition 3. Any allocation that satisfies the resource constraint and the enforcement constraints and has high implied interest rates is the outcome of a sustainable equilibrium for some choice of initial assets.

The immediate corollary to this proposition is the following.

Corollary. The constrained efficient allocations are the best sustainable equilibrium outcomes in the sense that they maximize (3) over the set of sustainable equilibrium outcomes.

So far we have presumed that the welfare weights in (3) are given and that in the decentralization we can set the initial assets. In some interpretations, we might take the initial assets as given and then figure out for what set of welfare weights the corollary holds. To do so, we can use a variant of the Negishi [18] and Mantel [16] algorithm that finds these relative weights in a fixed-point problem.

3. ADDING CAPITAL

We now explore how our results change when we move from a pure exchange economy to a growth model with capital. We first show in a constrained efficient allocation that if the enforcement constraints bind, then the Euler equation for capital is distorted. This result implies that a competitive equilibrium with government default alone cannot decentralize such an allocation. But if we give the government an extra instrument, a capital income tax, then the constrained efficient allocations can be decentralized.

3.1. A Growth Model

We modify our pure exchange economy in several ways. The preferences are the same as before. The resource constraints are now

$$c_{1t} + c_{2t} + k_{1t+1} + k_{2t+1} = A_{1t} f(k_{1t}) + A_{2t} f(k_{2t}) + (1-\delta)(k_{1t} + k_{2t}) \tag{29}$$

with k_{i0} given, where k_{it+1} is the capital stock chosen in period t for use in production in period $t+1$; $f(k)$ is a standard production function that is increasing, concave, and continuously differentiable and satisfies the standard Inada conditions; A_{it} is country-specific deterministically-fluctuating productivity; and δ is the depreciation rate of capital. The enforcement constraints are now

$$\sum_{s=t}^{\infty} \beta^{s-t} U(c_{is}) \geq V_{it}(k_{it}) \tag{30}$$

where

$$V_{it}(k_{it}) = \max \sum_{s=t}^{\infty} \beta^{s-t} U(c_{is}) \tag{31}$$

subject to

$$c_{it} + k_{it+1} = A_{it} f(k_{it}) + (1-\delta) k_{it}. \tag{32}$$

Notice that the problem with (financial) autarky reduces to that of a planning problem of a closed-economy growth model. Notice also that the value of utility under autarky in period t depends on the amount of capital located in country i in that period, k_{it}. The derivative of this value, $V'(k_{it})$, will be part of the root problem behind why the equilibrium with debt constraints alone cannot decentralize the constrained efficient allocations.

The constrained efficient allocations of this economy solve the planning problem of maximizing a weighted sum of the discounted utilities:

$$\max\left[\lambda_1\sum_{t=0}^{\infty}\beta^t U(c_{1t})+\lambda_2\sum_{t=0}^{\infty}\beta^t U(c_{2t})\right] \tag{33}$$

subject to the resource constraints (29) and the enforcement constraints (30) for country $i = 1, 2$ and all periods t, where λ_1 and λ_2 are nonnegative initial weights on the two countries' utilities.

An allocation $\{c_{1t}, c_{2t}, k_{1t+1}, k_{2t+1}\}_{t=0}^{\infty}$ is *constrained efficient* if it solves the planning problem for some nonnegative weights λ_1 and λ_2. Let $\beta^t\mu_{it}$ denote the multiplier on the enforcement constraints, and let γ_t denote the multiplier on the resource constraints. The first-order condition for consumption c_{it} is

$$\beta^t U'(c_{it})[\lambda_i+\mu_{i0}+\ldots+\mu_{it}]=\gamma_t \tag{34}$$

and the first-order condition for capital accumulation k_{it+1} is

$$\gamma_t+\mu_{it+1}V'_{it}(k_{it+1})=\gamma_{t+1}[f'(k_{it+1})+1-\delta]. \tag{35}$$

The transversality condition for capital accumulation is

$$\lim_{t\to\infty}\gamma_t k_{it+1}=0. \tag{36}$$

We can substitute (34) into (35) to get

$$U'(c_{it})=\beta U'(c_{it+1})[A_{it+1}f'(k_{it+1})+1-\delta]+\frac{\beta\mu_{it+1}}{M_{it}}(U'(c_{it+1})[A_{it+1}f'(k_{it+1})+1-\delta]-V'(k_{it+1}))$$

where $M_{it}=\lambda_{i0}+\mu_{i0}+\ldots+\mu_{it}$. From (34), we see that here, as in the pure exchange economy, an increase in consumption c_{it} in period t has two effects. One is the standard effect of raising the objective function by $\lambda_i\beta^t U'(c_{it})$. The other is to relax all of the incentive constraints from period 0 through period t. This effect has value $\beta^t U'(c_{it})(\mu_{i0} + \ldots + \mu_{it})$. Using (34), we see that the planner's effective intertemporal marginal rate of substitution between goods in period t and goods in period $t+1$ is

$$\frac{\gamma_{t+1}}{\gamma_t} = \frac{\beta U'(c_{it+1})}{U'(c_{it})} + \frac{\beta U'(c_{it+1})}{U'(c_{it})} \frac{\mu_{it+1}}{M_{it}}. \tag{37}$$

The first term on the right side of (37) is the standard one that arises from changing the value of the objective function by moving one unit of consumption from period t to period $t+1$. The second term on the right captures the following effect on incentives. Decreasing consumption in period t tightens the incentive constraints from period 0 to period t, while increasing consumption in period $t+1$ relaxes these constraints from period 0 to period $t+1$. The net effect on the incentive constraints by such a change is to relax the constraint in period $t+1$. The second term on the right side of (37) is nonnegative, and thus the incentive effect makes the planner have a higher intertemporal marginal rate of substitution than the standard one. By itself, this effect raises the benefit to having a higher capital stock at $t+1$ and pushes up the resulting investment.

From (35), we see that an increase in the capital stock in period t has, in addition to the standard effect of shifting resources from period t to period $t+1$, an effect on incentives captured by $\mu_{it+1} V'_{it+1}(k_{it+1})$. This term reflects the fact that by increasing the capital stock in $t+1$, the value of autarky $V_{it+1}(k_{it+1})$ is increased, and this tightens the incentive constraint on the margin by $V'_{it+1}(k_{it+1})$. This incentive effect, by itself, lowers the benefit to having a higher capital stock in $t+1$ and dampens the resulting investment.

To see how these various forces affect the Euler equation for capital, we can substitute (34) into (35) to get

$$\begin{aligned} & U'(c_{it}) - \beta U'(c_{it+1})[A_{it+1} f'(k_{it+1}) + 1 - \delta] \\ = \ & \frac{\beta \mu_{it+1}}{M_{it}} (U'(c_{it+1})[A_{it+1} f'(k_{it+1}) + 1 - \delta] - V'_{it+1}(k_{it+1})). \end{aligned} \tag{38}$$

If the incentive constraint for consumer i in period $t+1$ were slack (so that $\mu_{it+1} = 0$), then (38) would reduce to the familiar undistorted Euler equation for a growth model. The right side of (38) captures the two effects just discussed on this Euler equation.

As in the pure exchange economy we restrict ourself to constrained efficient allocations that satisfy a high implied interest rate condition. In this economy with capital the condition is

$$\sum_{t=0}^{\infty} q_{0,t}\left(A_{1t}f(k_{1t}) + A_{2t}f(k_{2t}) + (1-\delta)(k_{1t}+k_{2t})\right) < \infty. \tag{39}$$

where $q_{0,t} = q_{0,1}q_{1,2}\dots q_{t-1,t}$ and where $q_{t,t+1}$ is defined in (7). This condition guarantees that in the decentralized equilibrium the present value of gross output is finite. We use it guarantee that in our decentralization the constructed assets are finite and the transversality condition for bonds holds.

3.2. Decentralization With Government Default and Capital Income Taxes

Consider now decentralizing the constrained efficient outcome as a competitive equilibrium with taxes on capital income as well as government default on debt. With these two instruments, the government can mimic the distorted first-order conditions that define the constrained efficient outcome. The role of government default is the same as in the pure exchange economy. The role of capital income taxes is to make the consumers internalize the two effects on intertemporal effects on incentives that shifting consumption over time has.

The problem for a representative consumer in country 1 who faces both government debt default and capital income taxes is to maximize utility

$$\sum_{t=0}^{\infty} \beta^t U(c_{1t})$$

subject to the budget constraint

$$c_{1t} + p_{t,t+1}(s_{1t+1} - b_{1t+1}) + k_{1t+1} = w_{1t} + (1-\tau_{2t})s_{1t} - b_{1t} + R_{1t}k_{1t} + T_{1t} \tag{40}$$

and the nonnegativity constraints $s_{1t+1}, b_{1t+1} \geq 0$, with s_{10}, b_{10}, and k_{10} given. Here w_{1t} is the wage rate and $R_{it} = 1_t(1-\theta_{it})(r_{it}-\delta)$ is the return on capital after taxes and depreciation, where r_{it} is the before-tax return on capital and θ_{it} is the tax on capital income net of depreciation $(r_{it}-\delta)$. In this decentralization, there are firms which behave in a way we can summarize by conditions for rental rates and wage rates:

$$r_{it} = A_{it} f'(k_{it}) \text{ and } w_{it} = A_{it} f(k_{it}) - k_{it} A_{it} f'(k_{it}). \tag{41}$$

In this economy, a *competitive equilibrium with government debt default rates and capital income taxes* $\{\tau_{1t}, \tau_{2t}, \theta_{1t}, \theta_{2t}\}_{t=0}^{\infty}$ together with initial assets, liabilities, and capital stocks $\{s_{i0}, b_{i0}, k_{i0}\}_{i=1,2}$ consists of allocations $\{c_{1t}, c_{2t}, k_{1t+1}, k_{2t+1}\}_{t=0}^{\infty}$, assets $\{s_{1t+1}, s_{2t+1}\}_{t=0}^{\infty}$, liabilities $\{b_{1t+1}, b_{2t+1}\}_{t=0}^{\infty}$, and prices $\{p_{t,t+1}, r_{it}, w_{it}\}_{t=0}^{\infty}$ such that $\{c_{it}, s_{it+1}, b_{it+1}, k_{it+1}\}$ solves the consumer problem for each i and markets clear, so that $s_{1t+1} = b_{2t+1}$ and $b_{1t+1} = s_{2t+1}$ and the resource constraints (29) hold.

In this equilibrium, the first-order conditions for a consumer in country 1 are expressions (13)–(15) together with the Euler equation for capital

$$U'(c_{it}) = \beta U'(c_{it+1})(1 + (1 - \theta_{it+1})[A_{it+1} f'(k_{it+1}) - \delta]). \tag{42}$$

and the consumer's transversality condition for capital, namely

$$\lim_{t\to\infty} \beta^t U'(c_{it}) k_{it+1} = 0. \tag{43}$$

Proposition 4. Any allocation that satisfies the resource constraint and has high implied interest rates can be decentralized as a competitive equilibrium with government default and capital income taxes.

Proof. In our decentralization, we assume that the initial capital stock in each country, k_{10} and k_{20}, is owned by consumers in that country. In the competitive equilibrium, the initial physical capital stocks are given. The construction of the default rates, assets, liabilities, and prices is nearly identical to that for the pure exchange economy. As before, intertemporal prices $p_{t,t+1}$ are set by (8), and government default rates are set according to (16) and (17). We set rental rates and wage rates according to (41).

For assets and liabilities, we set

$$s_{it+1} - b_{it+1} = \frac{q_{t,t+1}}{p_{t,t+1}} \sum_{s=t+1}^{\infty} q_{t+1,s}(c_{is} - w_{is} + k_{is+1} - R_{is}k_{is}) \tag{44}$$

for $t \geq 0$ and for initial assets we set

$$s_{i0} - b_{i0} = \sum_{s=0}^{\infty} q_{0,s}(c_{is} - w_{is} + k_{is+1} - R_{is}k_{is}) \tag{45}$$

where we have set $R_{i0} = 1$. Notice that the high interest condition (equation 39) implies that right hand side of (44) and the right hand side of (45) are finite. If the right hand side of (44) is nonnegative, we set $b_{it+1} = 0$; if the right side of (44) is negative we set $s_{it+1} = 0$. We set s_{i0} and b_{i0} analogously from (45). Equation (44) defines the assets and liabilities chosen by consumers in equilibrium while (45) defines the initial assets and liabilities that are exogenously given to consumers.

For $t > 0$, the tax on capital income θ_{it} is backed out from the Euler equation

$$U'(c_{it}) = \beta U'(c_{it+1})[1 + (1 - \theta_{it+1})(A_{it+1}f'(k_{it+1}) - \delta)]$$

so that $R_{it+1} = [1 + (1 - \theta_{it+1})(A_{it+1}f'(k_{it+1}) - \delta)]$ is set equal to $U'(c_{it})/\beta \acute{U}'(c_{it+1})$. For $t = 0$ we set $R_{i0} = 1$.

To check the constructed assets and liabilities are budget feasible and that the transversality conditions for the individual are satisfied we substitute the budget constraint of the government $T_{1t} = \tau_{1t}b_{1t}$ into that of the consumer and use the same logic as in Proposition 1.

Finally, consider the transversality conditions. It is easy to adapt our earlier arguments to show that the transversality condition for bonds holds. The transversality condition in the constrained efficient allocation

$$\lim_{t \to \infty} \beta^t U'(c_{it})[\lambda_i + \mu_{i0} + \ldots + \mu_{it}]k_{it+1} = 0. \tag{46}$$

clearly implies that consumer's transversality condition for capital, namely

$$\lim_{t\to\infty} \beta^t U'(c_{it})k_{it+1} = 0 \tag{47}$$

since the sum of the multipliers $\mu_{i0} + \ldots + \mu_{it}$ is nonnegative. ■

3.3. Endogenizing the Default and Capital Income Tax Decisions

Here we briefly discuss how to endogenize the decisions of the governments in a dynamic game.

It is immediate to extend the *revert-to-autarky* plans considered in the pure exchange economy to the economy with capital. To do so, we let $\pi_t = (\pi_{1t}, \pi_{2t})$, where $\pi_{it} = (\tau_{it}, \theta_{it})$, and we let $x_t = (x_{1t}, x_{2t})$, with $x_{it} = (c_{it}, s_{it+1}, b_{it+1}, k_{it+1})$. It is straightforward to prove the analog of part of Proposition 2.

Proposition 5. In the economy with capital, an arbitrary triple of sequences (π, x, p) can be sustained by the revert-to-autarky plans if the triple is a competitive equilibrium with government default and capital income taxes for some choice of initial assets and if, for $i = 1, 2$ for every t, the following inequality holds:

$$\sum_{s=t}^{\infty} \beta^{s-t} U(c_{is}) \geq V_{it}(k_{it+1}). \tag{48}$$

It is then immediate to interpret the constrained efficient outcomes as the outcomes of a sustainable equilibrium with some suitably chosen initial assets. It remains an open question as to whether there can be equilibria with lower values than that of autarky. In particular, there may be equilibria in which consumers' expectations of future capital income taxes respond to the current government policies in a complicated way that is self-sustaining.

4. ADDING UNCERTAINTY

Throughout the paper we have focused on a deterministic economy in order to economize on notation, but all our results immediately generalize to a stochastic economy where the productivity A_{it} is a random variable. Constrained efficient allocations in this economy are characterized in Kehoe and Perri [12]. Note that in an economy without uncertainty the presence of limited enforcement limits the extent of intertemporal consumption smoothing while in an economy with uncertainty limited enforcement it limits both the extent of intertemporal smoothing and international risk sharing.

In each period t, the world economy experiences one of finitely many events $s_{t.}$ We denote by $s^t = (s_0, \ldots, s_t)$ the history of events up through and including period t. The probability, as of period 0, of any particular history s^t is $\pi(s^t)$. The initial realization s_0 is given, so that $\pi(s_0) = 1$. In each period t, the single good is produced in country i using inputs of capital $k_i(s^{t-1})$ and domestic labor $l_i(s^t)$. Production is also subject to a country-specific random shock $A_i(s^t)$, which follows an exogenous process. Output in country i at s^t is given by $F(k_i(s^{t-1}), A_i(s^t)l_i(s^t))$where F is a standard constant returns to scale production function. Consumers in country i have utility of the form

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi(s^t)U(c_i(s^t), l_i(s^t)) \tag{49}$$

where $c_i(s^t)$ denotes consumption by consumers in country i at s^t and β denotes the discount factor. The resource constraints are given by

$$\sum_{i=1,2}\left[c_i(s^t) + k_i(s^t)\right] = \sum_{i=1,2}\left[F(k_i(s^{t-1}), A_i(s^t)l_i(s^t)) + (1-\delta)k_i(s^{t-1})\right] \tag{50}$$

where δ is the per period depreciation rate of capital.

These enforcement constraints are of the form

$$\sum_{r=t}^{\infty}\sum_{s^r}\beta^{r-t}\pi(s^r|s^t)U(c_i(s^r), l_i(s^r)) \geq V_i(k_i(s^{t-1}), s^t) \tag{51}$$

where $\pi(s^r|s^t)$ denotes the conditional probability of s^r given s^t, $\pi(s^t|s^t) = 1$, and $V_i(k_i(s^{t-1}), s^t)$ denotes the value of autarky from s^t onward, which is given by the value of utility in the problem of choosing $k_i(s^r), l_i(s^r)$ and $c_i(s^r)$ for all s^r with $r \geq t$ to solve

$$V_i(k_i(s^{t-1}), s^t) = \max\sum_{r=t}^{\infty}\sum_{s^r}\beta^t\pi(s^r|s^t)U(c_i(s^r), l_i(s^r))$$

subject to

$$c_i(s^r) + k_i(s^r) \leq F(k_i(s^{r-1}), A_i(s^r)l_i(s^r)) + (1-\delta)k_i(s^{r-1})$$

with $k_i(s^{t-1})$ given.

The analysis is nearly identical to that in the deterministic economy. The only interesting point is that there is a number of degrees of freedom in assigning the state-contingent capital income taxes that decentralize the constrained efficient allocations. To see this note that in the decentralized allocation the first order condition for capital is

$$U_{ic}(s^t) = \beta \sum \pi(s^{t+1}|s^t)U_{ic}(s^{t+1})[1 + (1 - \theta_i(s^{t+1})(A_i(s^{t+1})F_{ik}(s^{t+1}) - \delta)] \tag{52}$$

Let $\theta_i(s^t, s_{t+1})$ satisfy (52) at the constrained efficient allocations. Then if $\hat{\theta}_i(s^t, s_{t+1})$ satisfies

$$\sum \pi(s^{t+1}|s^t)U_{ic}(s^{t+1})[\theta_i(s^{t+1})(A_i(s^{t+1})F_{ik}(s^{t+1})] = \sum \pi(s^{t+1}|s^t)U_{ic}(s^{t+1})[\hat{\theta}_i(s^{t+1})(A_i(s^{t+1})F_{ik}(s^{t+1})]$$

then $\hat{\theta}_i(s^t, s_{t+1})$ also decentralizes this allocation. One way to uniquely assign such taxes is to suppose that they do not vary with the current state so that $\theta_i(s^t, s_{t+1}) = \bar{\theta}(s^t)$.

5. CONCLUSION

We have proposed a decentralization of constrained efficient allocations in which the forces that produce the limited risk-sharing are more explicitly modeled than in the existing literature. The decentralization is intuitively appealing when applied to international risk-sharing problems for economies with capital and a limited ability to enforce contracts. It may be possible to similarly model the forces that limit risk-sharing in other decentralizations, for example, an equilibrium in which the debt constraints studied by Alvarez and Jermann [2] are explicitly chosen by financial intermediaries in an appropriately defined dynamic game.

Here we have focused on a deterministic economy in order to economize on notation, but all our results immediately generalize to a stochastic economy, provided that debt constraints, capital constraints, and taxes can be state-contingent.

Notes

[1]Much of the work on sovereign debt is motivated by the debt crises of the 1970s and the 1980s. In discussing these crises Eaton and Fernandez (1995, p. 2059) argue the following

"Most of the debt that developing countries ran up during the 1970s and the 1980s was incurred or guaranteed by the governments of these countries. One reason for the prominent role of the government might have been creditors' suspicions about the local judicial system's ability or willingness to enforce a loan contract with a private debtor. Even in cases where debt was initially nonguaranteed, private creditors turned to the government to make good on loans that went sour."

Eaton and Fernandez (1995, p2059) discuss cases in which even when debts seemed to be completely private, foreign creditors held the government accountable for assuming the obligations of its citizens whenever these citizens did not pay. The events of the Argentine debt crisis in November of 2001 seem also to confirm this pattern continues to the present. Through the imposition of capital controls and banking restrictions the Argentinian government has de facto forced private debtors to (partially) default on their foreign, dollar denominated, debt.

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