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## **Accounting for the Great Depression\***

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ABSTRACT \_\_\_\_\_

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The Great Depression is arguably the most significant economic event of the 20th century. Bordo, Erceg, and Evans (2001) develop a model in which sticky wages combine with a monetary contraction to produce a downturn in output. Cole and Ohanian (2001) argue that the slow recovery is due to increased cartelization and unionization. Bernanke and Gertler (1989) and Cooper and Ejarque (2000) develop models in which monetary contractions increase frictions in capital markets and lead to downturns driven by collapses in investment. In our view the critical feature of the sticky wage story or the cartelization story is that both lead to a wedge between marginal rates of substitution between consumption and leisure and the marginal product of labor. The critical feature of the capital market story is that capital market frictions introduce a wedge between the intertemporal marginal rate of substitution in consumption and the marginal product of capital.

We show that the aggregate properties of these models are the same as those of a prototype growth model with time-varying taxes on labor and investment chosen to reproduce the wedges generated by the original models. We also show that models with frictions in financing inputs leads to a wedge between inputs and outputs. Such models have the same aggregate properties as a growth model with time-varying productivity. (See Bergoing et al. 2002 for other frictions that show up as time-varying productivity.)

These observations lead us to conclude that a large class of business cycle models are equivalent to a prototype growth model with time varying productivity, labor taxes, and investment taxes. The time-varying productivity capture the *goods market distortions* in the original economy. The labor taxes and the investment taxes capture the *labor market distortions* and the *capital market distortions* in the original economy. We use a parameterized prototype growth model to measure these distortions in the data from 1929 to 1939. We

then feed these distortions back into the growth model to assess what fraction of the output movements can be attributed to each. We find that most of the decline in output from 1929 to 1933 is due to goods market distortions while much of the slow recovery from 1933 to 1939 is due to labor market distortions. Investment distortions play, at best, a minor role. (See Cole and Ohanian 1999 for some related conclusions.)

The goal of this *business cycle accounting* is to guide researchers to focus on models with the kinds of frictions that can deliver the quantitatively relevant types of observed distortions in the prototype economy.

## 1. Equivalence Results

Here we show that an original economy with input financing frictions has the same aggregate properties as a prototype economy with goods market distortions which look like time-varying productivity. In Chari, Kehoe and McGrattan we show that sticky wages or cartels and unions show up labor market distortions while capital market frictions show up as investment distortions.

### A. Goods Market Distortions from Input Financing Frictions

Here we consider a simple economy with distortions in the allocation of inputs across two types of firms arising from financing frictions. Both types of firms must borrow in advance of production to pay for an input, say labor. The first type of firms are financially constrained in the sense that they pay a higher price for borrowing than the second type. We think of these frictions as capturing the idea that some firms, often thought of as small firms, find it difficult to finance borrowing. One source of the higher price paid by the financially constrained firms is that moral hazard problems are more severe for small firms.

For simplicity we consider a one period economy in which firms borrow at the beginning of the period to finance inputs and repay their borrowings at the end of the period. Final output  $y$  is produced from the outputs of sectors 1 and 2, namely  $y_1$  and  $y_2$  according to  $y = y_1^\gamma y_2^{1-\gamma}$ . The representative firm producing final output solves

$$\max_{y_1, y_2} y_1^\gamma y_2^{1-\gamma} - p_1 y_1 - p_2 y_2$$

where  $p_i$  is the price of output of sector  $i$ . Firms in sector  $i$  hire labor  $l_i$  to produce output according to  $y_i = l_i^\alpha$  and solve  $\max p_i l_i^\alpha - R_i w l_i$  where  $w$  is the wage rate and  $R_i$  is the gross interest rate paid on borrowing by firms in sector  $i$ . We imagine that firms in sector 1 are more financially constrained than those in sector 2 so that  $R_1 > R_2$ . Let  $R_i = R(1 + \tau_i)$  where  $R$  is the rate savers earn and  $\tau_i$  measures the wedge between the rate paid to savers and the rate paid by borrowers in sector  $i$  induced by financing constraints. Since savers do not discount utility within the period  $R = 1$ .

Consumers choose consumption  $c$  and labor  $l$  to maximize  $U(c, l)$  subject to  $c = wl + \Pi$  where  $\Pi$  are the profits earned by firms. The resource constraints are  $l = l_1 + l_2$  and  $c = y$ .

Now consider a prototype economy with a single sector and an aggregate production function  $y = Al^\alpha$  with productivity parameter  $A$ . The representative firm maximizes  $Al^\alpha - wl$  where  $\tau$  and consumers maximize  $U(c, l)$  subject to  $c = (1 - \tau)wl + \Pi$ . Here  $1 - \tau$  is the *labor market distortion*, namely the wedge between the marginal product of labor and the marginal rate of substitution between leisure and consumption. In Chari, Kehoe, McGrattan (2002), we prove the following proposition.

*Proposition 1.* Consider an equilibrium allocation and a wage rate for the economy

with input financing frictions. These allocations coincide with those of the prototype economy with technology  $A$  and labor tax rate  $\tau$  where

$$A = \frac{[\gamma(1 + \tau_2)]^{\alpha\gamma} [(1 - \gamma)(1 + \tau_1)]^{\alpha(1-\gamma)}}{[(1 - \gamma)(1 + \tau_1) + \gamma(1 + \tau_2)]^\alpha} \quad (1)$$

and

$$1 - \tau = \frac{\gamma}{1 + \tau_1} + \frac{1 - \gamma}{1 + \tau_2}. \quad (2)$$

Consider a repeated version of this economy in which  $\tau_1$  and  $\tau_2$  fluctuate over time. Suppose that these fluctuations are such that the right side of (2) is constant. Thus, on average, financing frictions are unchanged but relative frictions fluctuate. An outside observer who attempted to fit the data generated by the economy with input financing frictions using the prototype economy would identify the fluctuations in relative distortions with fluctuations in technology and would see no fluctuations in the labor market distortion. In particular, periods in which the relative distortions increase would be misinterpreted as periods of technological regress. This observation leads us to label  $A$  as the *goods markets distortion* in the prototype economy.

More generally, fluctuations in the input financing wedges  $\tau_1$  and  $\tau_2$  which lead to fluctuations in  $\tau$  would show up in the prototype economy in fluctuations in both the goods market distortion  $A$  and the labor market distortion  $\tau$ .

## B. Labor Market Distortions from Unions and Cartels

Consider the following cartelized economy in which fluctuations in policies towards unions and cartels show up as fluctuations in labor market distortions in the prototype econ-

omy. (See Cole and Ohanian 2001 for a discussion of such policies in the Great Depression. Sticky wages lead to similar distortions.)

For simplicity, we consider a single period model in which a final good is produced from a continuum of intermediate goods according to

$$y = \left[ \int y(i)^{1/\rho} di \right]^\rho \quad (3)$$

where intermediate goods are produced according to  $y(i) = l(i)^{1-\alpha}$  and labor services  $l(i)$  are produced from a continuum of types of labor according to

$$l(i) = \left[ \int l(i, j)^{1/v} dj \right]^v. \quad (4)$$

We imagine that each intermediate good  $i$  is produced by a large number of firms referred to as industry  $i$ . We imagine that firms in this industry act collusively to maximize industry profits. Each union  $j$  represents all the households who supply labor of type  $j$ . Unions are monopolists and set a wage rate  $w(j)$  for labor of type  $j$ . Final goods producers are competitive and solve

$$\max py - \int p(i)y(i)di$$

subject to (3). The demand functions are  $y^d(i) = [p/p(i)]^{\frac{\rho}{\rho-1}} y$ . Industry  $i$  chooses  $p(i)$ ,  $l(i, j)$  to solve

$$\max p(i)y(i) - \int w(j)l(i, j)dj$$

subject to (4), and  $y(i) = y^d(i)$ . This problem yields the demand for labor  $l(i, j)$  of the

form

$$l^d(i, j) = \left[ \frac{w}{w(j)} \right]^{\frac{v}{v-1}} l^d(i)$$

where  $w = [f(w(j))^{\frac{1}{1-v}} dj]^{1-v}$ . The total demand for type  $j$  labor is  $l^d(j) = \int l^d(i, j) di$ .

The problem of the  $j$ th union of households is to choose  $c(j)$  and  $w(j)$  to maximize  $U(c(j), l(j))$  subject to  $c(j) - w(j)l(j) = \Pi$  and  $l(j) = l^d(j)$ . In a symmetric equilibrium profit maximization by firms implies that  $p = \rho w / F_l$ . Thus the firm optimally sets the price as a (gross) markup  $\rho$  above marginal cost  $w / F_l$ . Similarly, unions optimally set the wage rate as a (gross) markup  $v$  over the marginal rate of substitution between leisure and consumption so that  $w = vp(-U_l / U_c)$ . Combining these gives

$$F_l = v\rho(-U_l / U_c). \quad (5)$$

Hence, imperfect competition adds the “wedge”  $v\rho$  between the marginal rates of substitution and transformation. The allocation is completely described by (5) and by the resource constraint  $c = l^{1-\alpha}$ .

We think of government pro-competitive policy as limiting the monopoly power of industries and unions putting pressure on these entities to limit their anti-competitive behavior. For example, the government policy is that if the markups exceed, say  $\bar{\rho} \leq \rho$  by an industry or  $\bar{v} \leq v$  by a union then the government will enforce provisions that make these entities price competitively. Under such a policy it is immediate that the markup charged by industries is  $\bar{\rho}$  and that charged by unions is  $\bar{v}$ . Under these policies the equilibrium is described by

$$F_l = \bar{v}\bar{\rho}(-U_l / U_c) \quad (6)$$

where the size of the wedge varies with the policy. Extreme enforcement is captured by having  $\bar{\rho} = \bar{v} = 1$  so that the economy reproduces the competitive outcome.

Consider a prototype economy in which consumers maximize  $U(c, l)$  subject to  $c = (1 - \tau)wl + T + \Pi$  where  $\tau$  is a labor market distortion,  $T$  is a lump sum rebate equal to  $\tau wl$ , and  $\Pi$  is profits. A representative firm maximizes  $l^{1-\alpha} - wl$  and the resource constraint is  $c = l^{1-\alpha}$ .

In Chari, Kehoe and McGrattan (2002) we prove

*Proposition 2.* Consider an allocation for the cartelized economy for some given government policy  $\{\bar{\rho}, \bar{v}\}$ . These allocations coincide with those of the prototype economy with tax distortions  $\{1 - \tau\}$  where  $1 - \tau = \bar{\rho}\bar{v}$ .

Consider a repeated version of the cartelized economy in which the government policy  $(\bar{\rho}, \bar{v})$  fluctuates over time. An outside observer using the prototype economy to fit the data of the cartelized economy would interpret fluctuations in government policies towards cartels and unions as fluctuations in labor market distortions.

### C. Capital Market Distortions from Investment Frictions

For investment frictions the link between the original economy and a prototype economy is immediate. Many of the frictions discussed in the literature end up affecting the economy by raising the cost of investment for firms. These frictions show up in the prototype economy as a tax on investment, raising the cost of investment from 1 to  $1 + \tau_x$ . We refer to  $1 + \tau_x$  as the *capital investment distortion*. In the prototype economy the intertemporal

Euler equation is of the form

$$(1 + \tau_{xt})U_{ct} = \beta U_{ct+1}[F_{kt+1} + (1 + \tau_{xt+1})(1 - \delta)]$$

where we have used the standard notation of the growth model. The equilibrium is then characterized by this Euler equation together with the labor market condition  $-U_{lt}/U_{ct} = F_{lt}$  and the resource constraint,  $c_t + k_{t+1} = F(k_t, l_t) + (1 - \delta)k_t$ .

## 2. A Prototype Economy

Motivated by our equivalence results we consider a prototype economy which is a deterministic growth model with 3 time varying parameters:  $A_t$ ,  $(1 - \tau_{nt})$  and  $(1 + \tau_{xt})$ . In this economy consumers maximize  $\sum_t \beta^t U(c_t, l_t)$  subject to

$$c_t + (1 + \tau_{xt})(k_{t+1} - (1 - \delta)k_t) = (1 - \tau_{nt})w_t l_t + r_t k_t + T_t.$$

where  $g_t$  is government consumption. Firms maximize  $A_t F(k_t, l_t) - r_t k_t - w_t l_t$ . The equilibrium is summarized by the resource constraint,  $c_t + k_{t+1} = y_t + (1 - \delta)k_t$  where

$$y_t = A_t F(k_t, l_t) \tag{7}$$

the labor market condition

$$-\frac{U_{lt}}{U_{ct}} = (1 - \tau_{nt})A_t F_{lt}, \tag{8}$$

the capital market condition

$$(1 + \tau_{xt})U_{ct} = \beta U_{ct+1}[(A_{t+1}F_{kt+1} + (1 + \tau_{xt+1})(1 - \delta)]. \quad (9)$$

In this economy  $A_t$  shows up as a technology parameter but given our equivalence results we interpret it as the *goods market distortions parameter*. Likewise we interpret  $(1 - \tau_{nt})$  as a *labor market distortions parameter* and  $1/(1 + \tau_{xt})$  as a *capital market distortions parameter*.

We assume preferences and technology are given by  $U(c, l) = \log c + \psi \log(1 - l)$  and  $F(k, l) = k^\alpha l^{1-\alpha}$ . We set  $\beta = .97$ ,  $\psi = 2.1$ ,  $\alpha = .36$ ,  $\delta = .048$ . In our experiments we make our prototype economy more comparable to the data by adding constant government spending equal to 10.7 percent of 1929 GDP, constant growth in population and productivity of 1.5% and 1.6% per year, and a constant tax rate on capital income of 39%. (For details, see Chari, Kehoe, McGrattan 2002.)

Given data on  $y_t, k_t, l_t$  and  $c_t$ , we use (7), (8) and (9) to construct series for goods, labor and capital market distortions from 1929 to 1939. We set  $1 + \tau_x$  to be 1 in 1929. In Figure 1 we plot the resulting series for these distortions as well as real GNP after removing a 3.1% trend growth rate and a 1.6% trend in  $A_t$ . We normalize all the series to be 100 in 1929. In the data, GNP is 38% below trend in 1933 and by 1939 is still 25% below trend. In 1933  $A_t$  is 22% below trend, but by 1939 it had essentially recovered to trend. In 1933,  $1 - \tau_{nt}$  is 23% lower than its 1929 level and in 1939 is still 26% below its 1929 level. Under our interpretation, goods and labor markets become substantially more distorted from 1929 to 1933. By 1939, the added distortions in the goods market disappeared but that labor

markets remained as distorted as they were in 1933. (This conclusion is consistent with the analysis of Cole and Ohanian 1999.)

Finally, note from Figure 1b that  $1/(1 + \tau_x)$  is higher than its 1929 level throughout the 1930's. Under our interpretation, Figure 1b implies that capital markets actually became *less* distorted in the Great Depression. We find this conclusion implausible. Our procedure for measuring distortions is questionable when it comes to capital markets distortions because we have abstracted from uncertainty. In a growth model with uncertainty the right side of (9) would be replaced by its expected value. In such a case a procedure for uncovering the capital market distortion would require specifying a joint stochastic process on all variables in that equation. We propose an alternative simpler method for assessing the potential importance of investment distortions.

### **A. The Prototype Economy with Goods and Labor Market Distortions**

Here we use our constructed measures of goods and labor market distortions to ask, What fraction of output fluctuations can be accounted for by these distortions? We answer this question by simulating our prototype economy with our constructed measures of distortions.

In our *goods distortions experiment* we use our constructed goods distortion parameter and set the other distortions to their 1929 level, while in our *labor distortions experiment* we use our constructed labor distortions parameter and set the other distortions to their 1929 level. In Figure 2A,2B and 2C we compare the results of our experiments with the data for output, labor and investment. For the goods distortion experiment we find that that prototype economy generates much of the observed downturn in output but it generates

much too rapid a recovery. For example, by 1933 output in the model falls about 27% in the model and by about 38% in the data. By 1939 the model generates only 8% of the observed 25% downturn in output. As can be seen in Figure 2B, the reason for this rapid recovery in output is that prototype economy completely misses the continued sluggishness in labor from 1933 onwards. In terms of investment the prototype economy shows a larger fall from 1929 to 1933 than in the data. From 1933 onward, investment recovers more rapidly in the prototype economy than in the data.

Relative to the data, our economy with only labor distortions generates much too small a decline in output and investment but mirrors the patterns of labor fairly well. In terms of output, by 1933 output in the model falls by only 10% while in the data it falls by 38%. By 1939 output in the model output in the model falls by 14% while in the data it falls by 25%. In terms of labor, the labor distortions generate the sluggishness in labor input seen after 1933.

These observations suggest that a prototype economy with both goods and labor distortions might be able to account for the magnitude of the decline in output and the slow recovery. In our *goods and labor distortions experiment* we use our constructed distortion parameters for goods and labor markets and set the capital market distortion parameter to its 1929 level. In Figure 3 we see that the model captures both the downturn in output and the slow recovery remarkably well. It also generates the sluggishness in labor after 1933. The main weakness is that it generates too sharp a fall and too rapid a recovery in investment.

## B. The Prototype Economy with Investment Distortions

We are interested in asking, What fraction of output fluctuations can be accounted for by investment distortions? Given the difficulties we experienced in inferring a reasonable level of investment distortions from the data we are wary of trying to answer this question by simply putting in the distortions  $1/(1 + \tau_x)$  inferred from (9). Instead we do the following. We consider a prototype economy with the goods and labor market distortions set to their 1929 levels and let the investment distortions be whatever they have to be so as to generate the actual investment series. In a sense, by attributing all movements in investment to these distortions we feel that this *investment distortions experiment* if anything overstates the fraction of output fluctuations that can be attributed to such distortions.

In Figures 4A and 4B we see that the prototype economy with investment distortions generates only modest fall in output from 1929 to 1933 and does not generate the recovery after 1933. While it does generate a recovery in labor, the effect on output is offset by the cumulative effect of the decade long investment slump.

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## Appendix A : Algebra for Propostion 1

Final goods firm

$$\max_{y_1, y_2} y_1^\gamma y_2^{1-\gamma} - p_1 y_1 - p_2 y_2$$

with first order conditions

$$\gamma y = p_1 y_1 \text{ and } (1 - \gamma)y = p_2 y_2 \tag{10}$$

Sector  $i$ ,  $\max p_i l_i^\alpha - R_i w l_i$  with first order conditions

$$(1 + \tau_i)w = \alpha \frac{p_i y_i}{l_i} \tag{11}$$

Substitute equations from (10) into (11) to get

$$\alpha \frac{\gamma y}{l_1} = (1 + \tau_1)w \text{ and } \alpha \frac{(1 - \gamma)y}{l_2} = (1 + \tau_2)w$$

So  $l_2/l_1 = \frac{1-\gamma}{\gamma} \frac{1+\tau_2}{1+\tau_1}$  so using  $l_1 + l_2 = l$  gives

$$l_1 = \frac{\gamma(1 + \tau_2)}{(1 - \gamma)(1 + \tau_1) + \gamma(1 + \tau_2)} \quad (12)$$

$$l_2 = \frac{(1 - \gamma)(1 + \tau_1)}{(1 - \gamma)(1 + \tau_1) + \gamma(1 + \tau_2)} \quad (13)$$

Plug these into  $y = (l_1^\alpha)^\gamma (l_2^\alpha)^{1-\gamma}$  to get formula for  $A$ . To get formula for  $(1 - \tau)$  note that in the prototype economy the first order condition reduces to

$$\frac{w}{\alpha l^{\alpha-1}} = (1 - \tau)A \quad (14)$$

while in the original economy use  $w = \alpha \gamma y / [(1 + \tau_1)l_1]$  with  $y = Al^\alpha$  and (12) to conclude

$$\frac{w}{\alpha l^{\alpha-1}} = \frac{\gamma A}{(1 + \tau_1)} \frac{(1 - \gamma)(1 + \tau_1) + \gamma(1 + \tau_2)}{\gamma(1 + \tau_2)} \quad (15)$$

If these two economies are to be the same the right sides of (14) and (15) must be the same so that

$$(1 - \tau) = \frac{\gamma}{1 + \tau_1} + \frac{1 - \gamma}{1 + \tau_2}$$

which is the expression in Proposition 1.

## **Appendix B: Equivalence for labor market distortions and capital market distortions**

Figure 1. GNP and Two Distortions

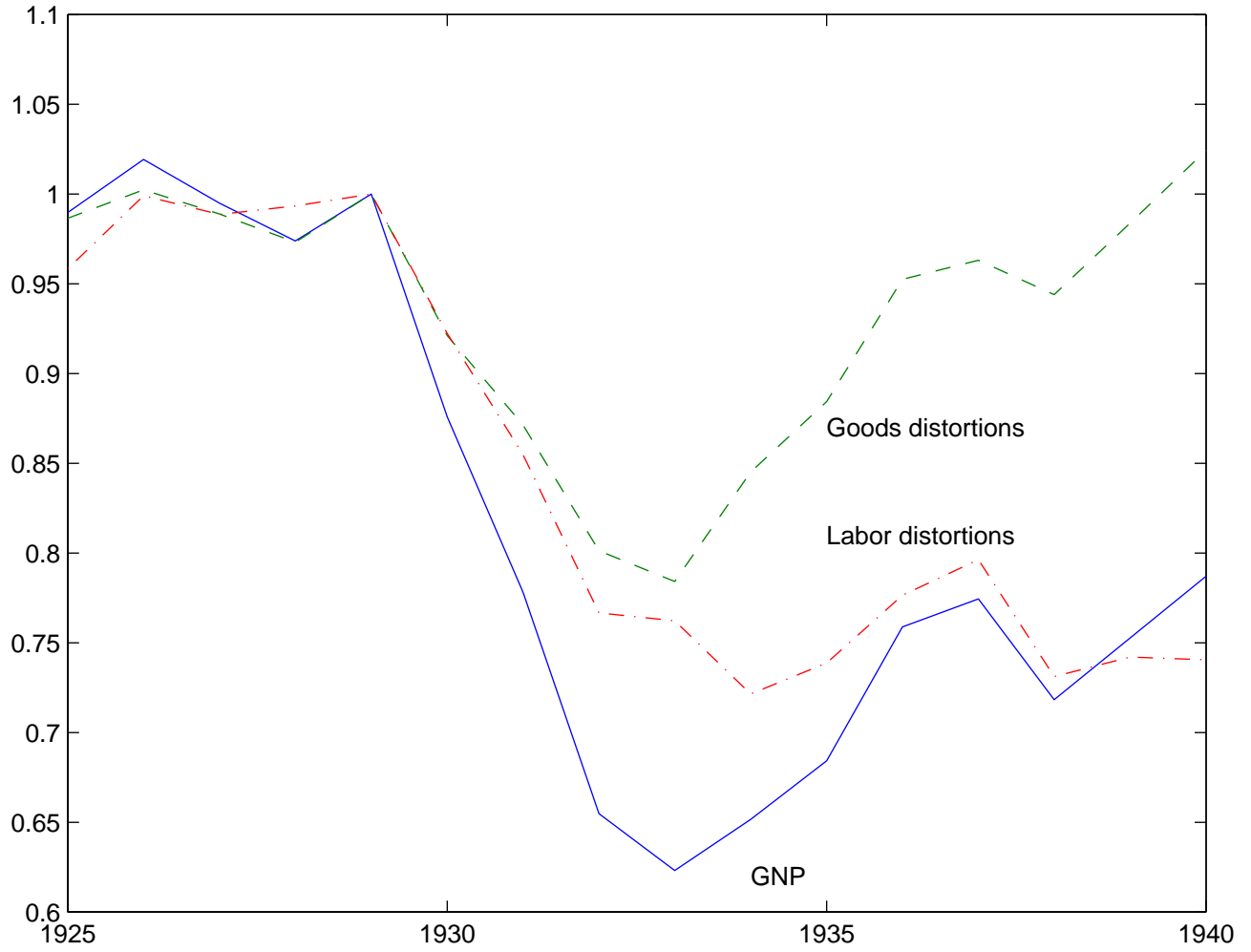


Figure 1b. GNP and Investment Distortions

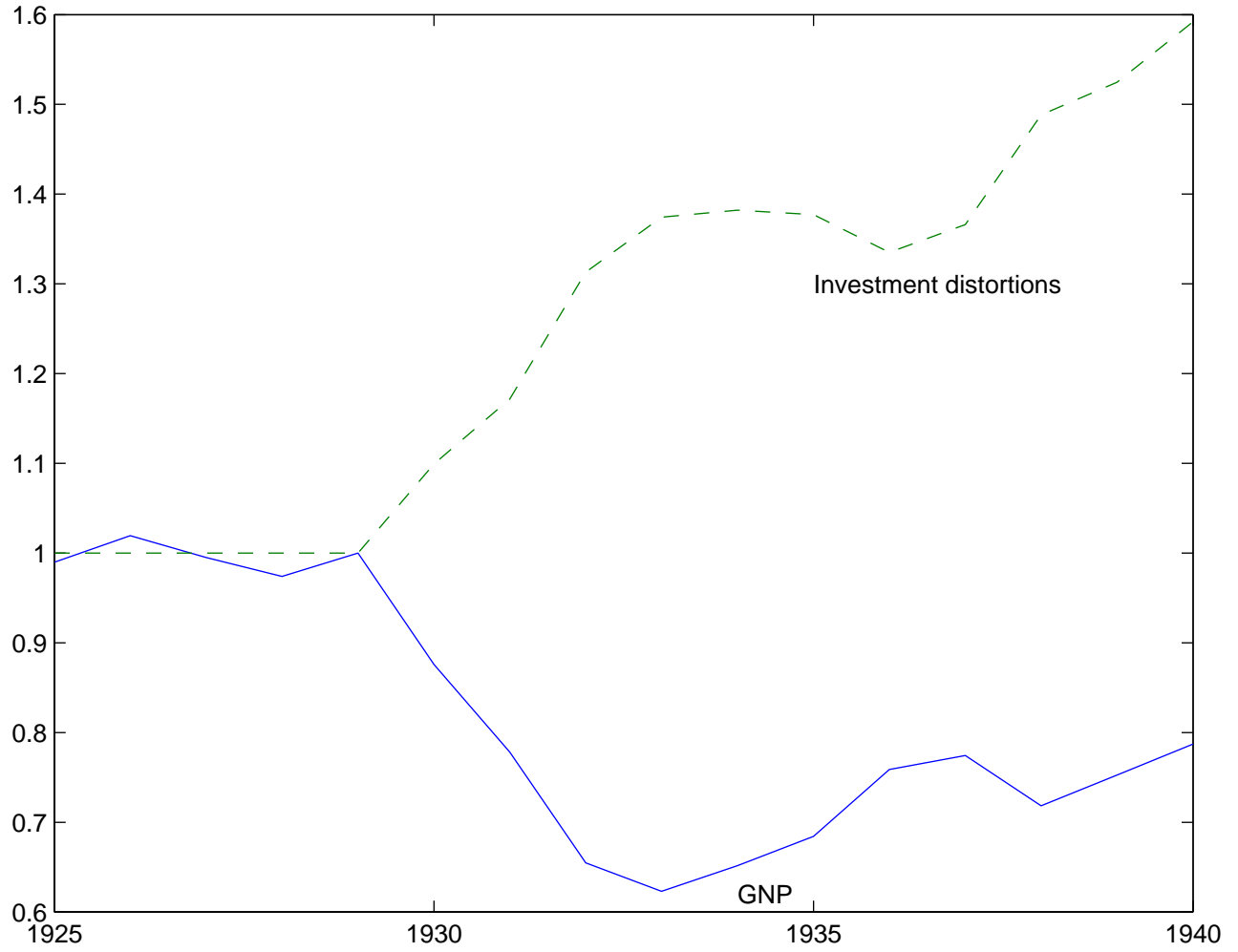


Figure 2a. GNP -- Actual and Simulated from Two Models

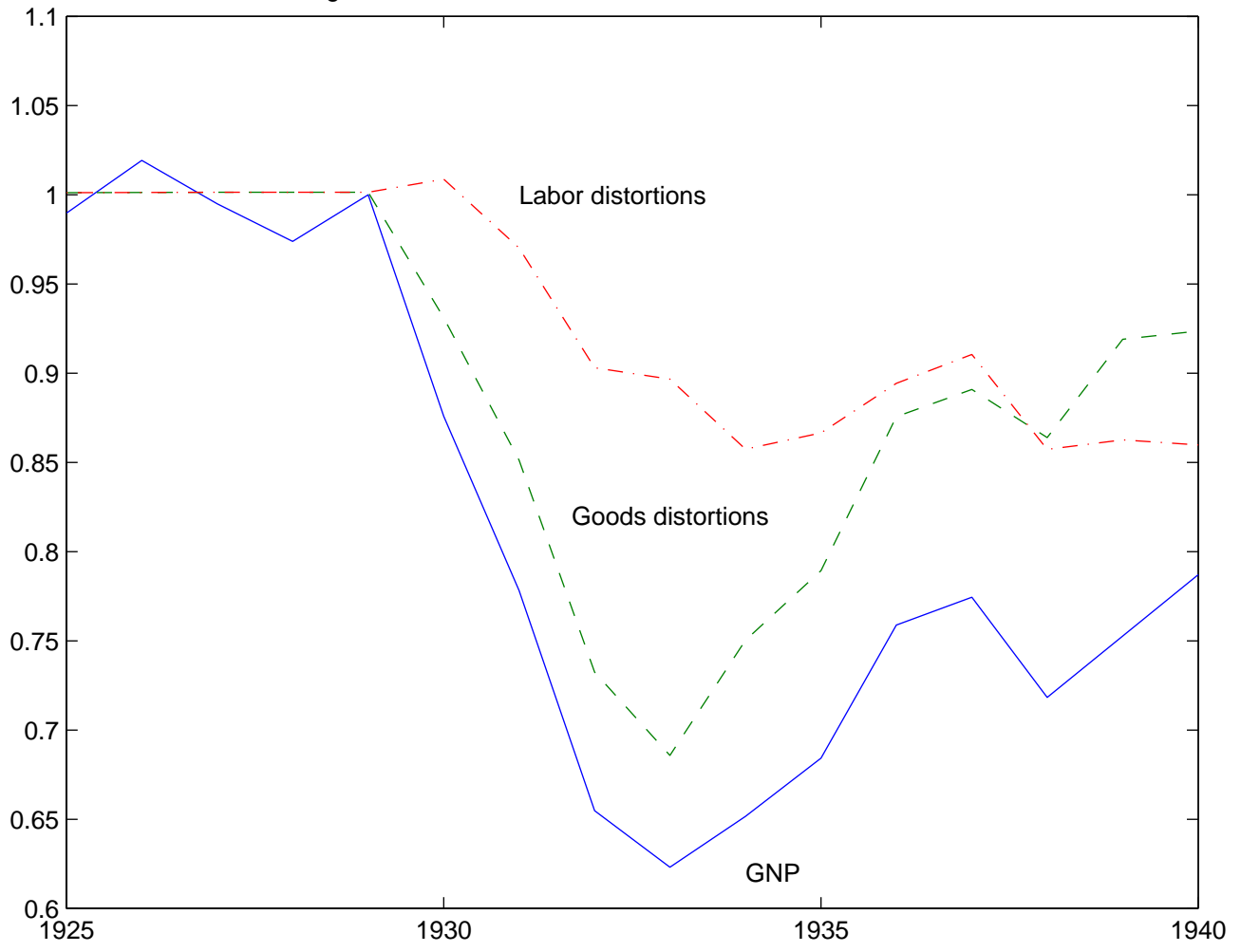


Figure 2b. Labor -- Actual and Simulated from Two Models

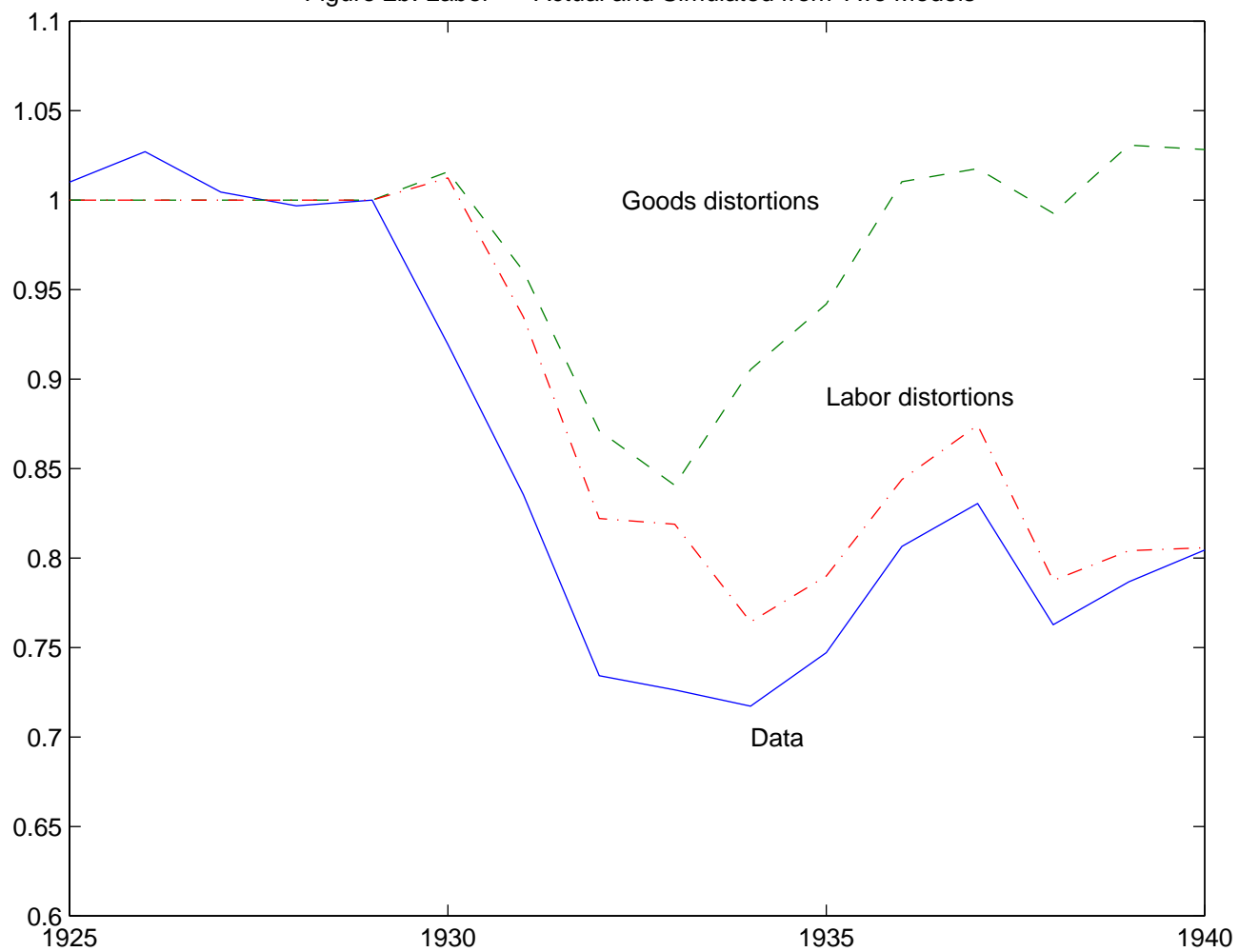


Figure 2c. Investment -- Actual and Simulated from Two Models

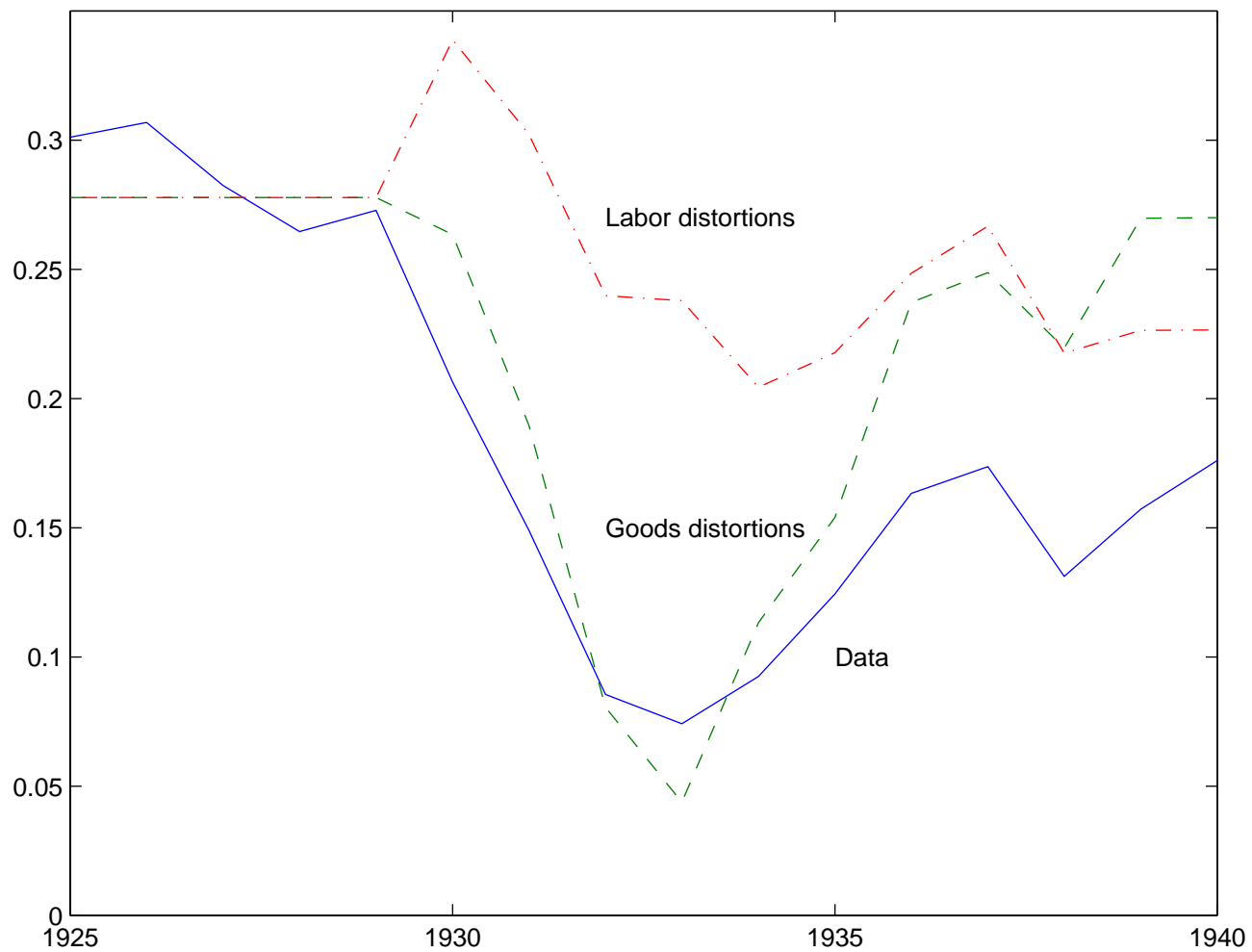


Figure 3a. GNP -- Actual and Simulated from Combined Model

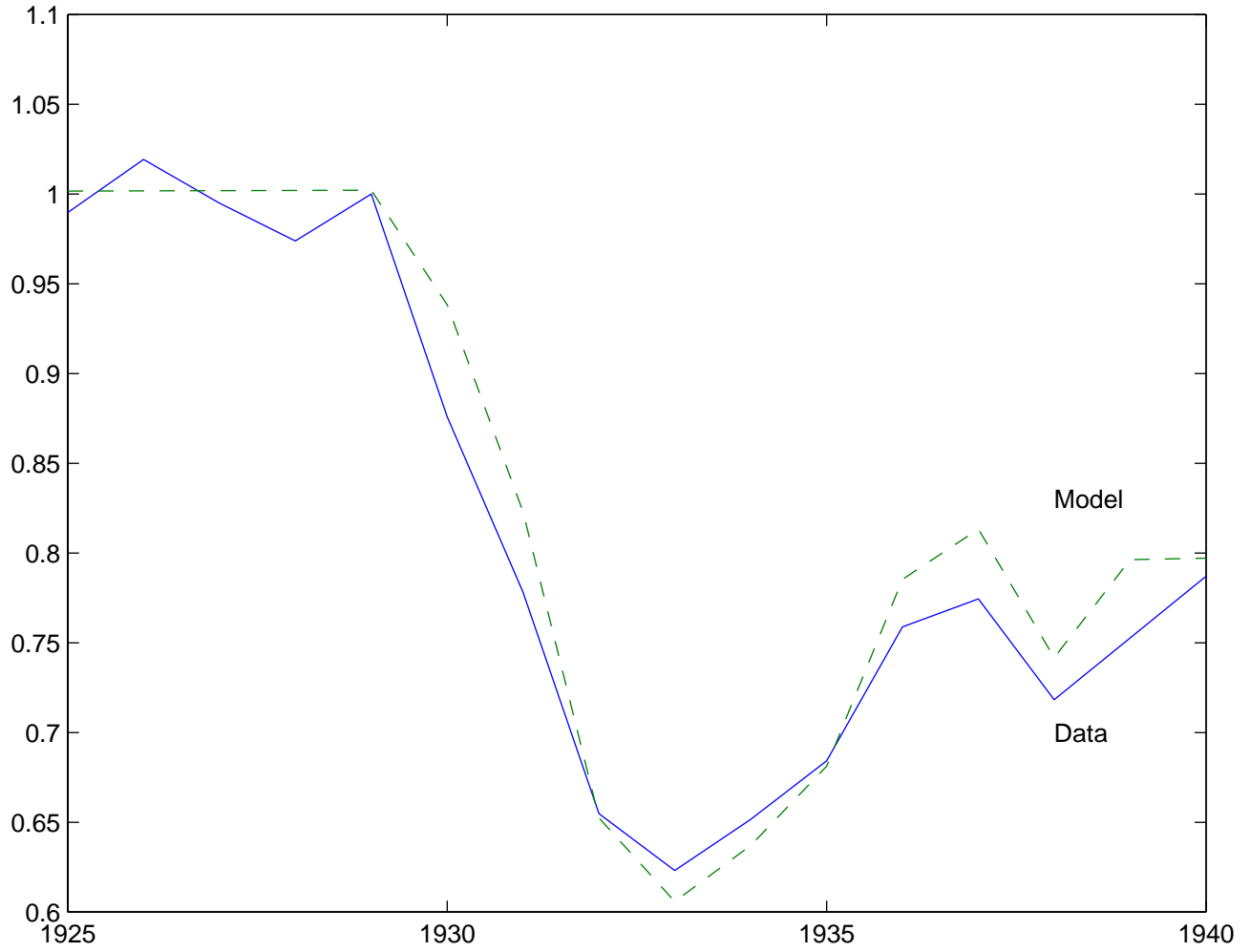


Figure 3b. Labor -- Actual and Simulated from Combined Model

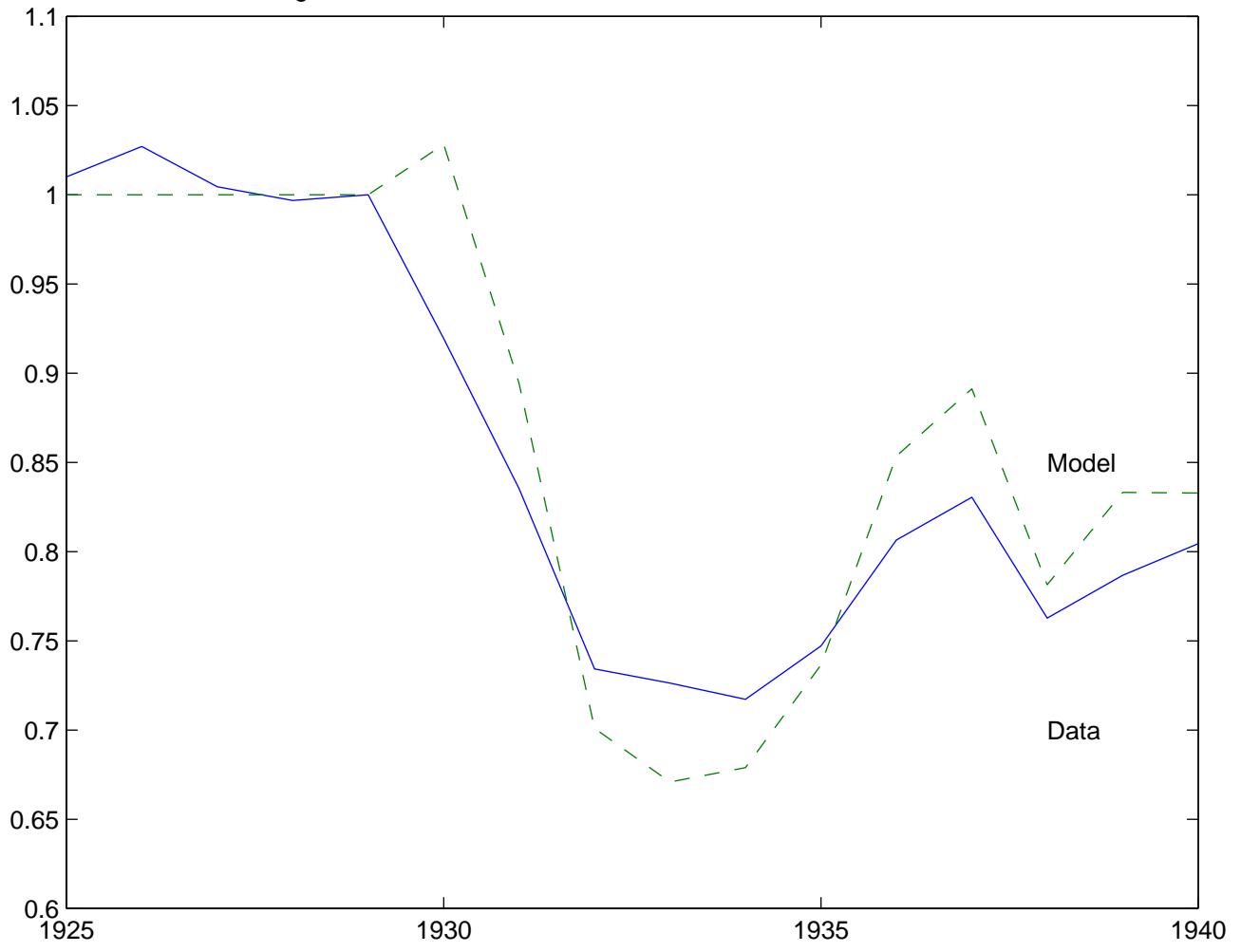


Figure 3c. Investment -- Actual and Simulated from Combined Model

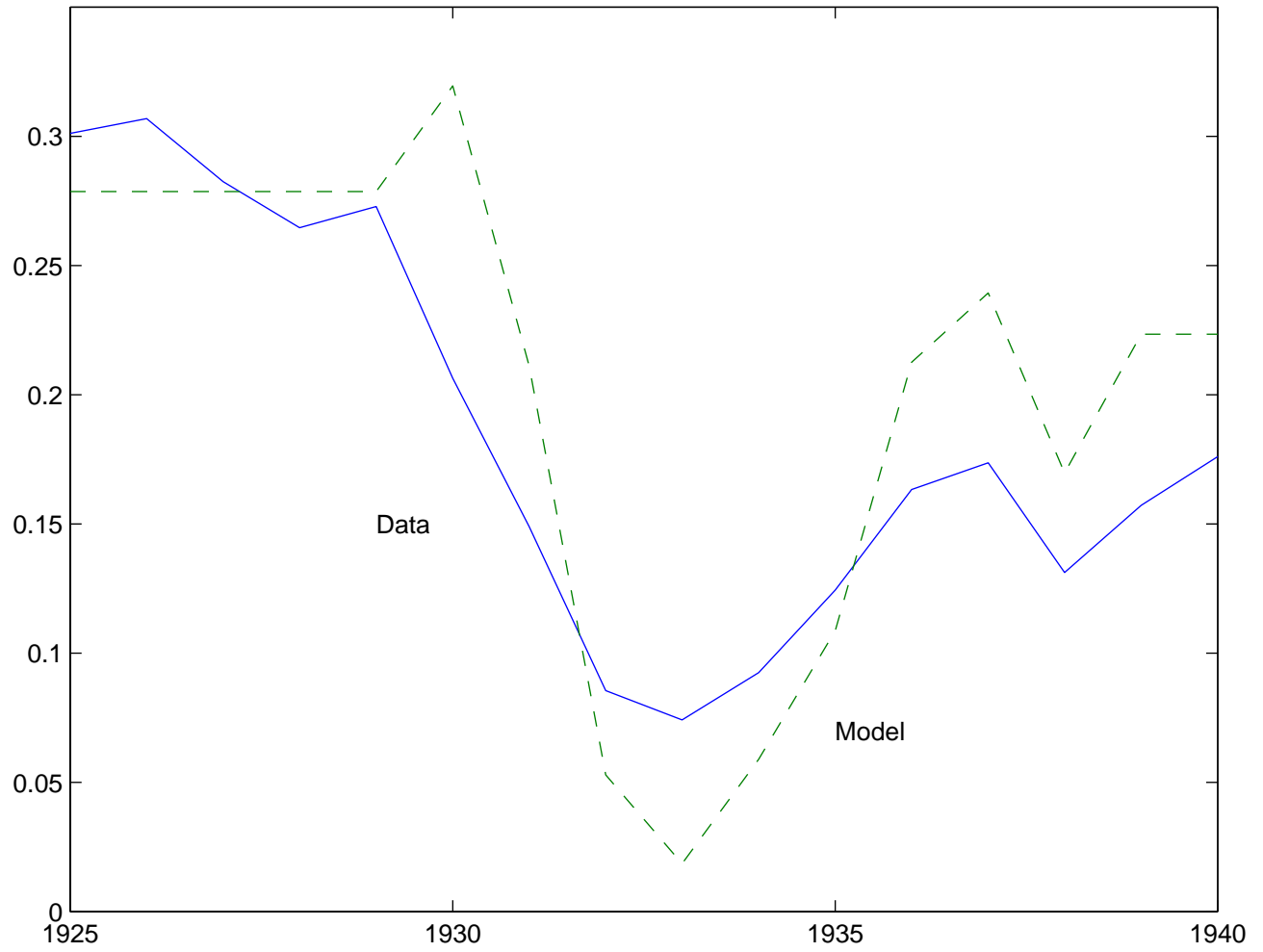


Figure 4a. GNP -- Actual and Simulated from Investment Model

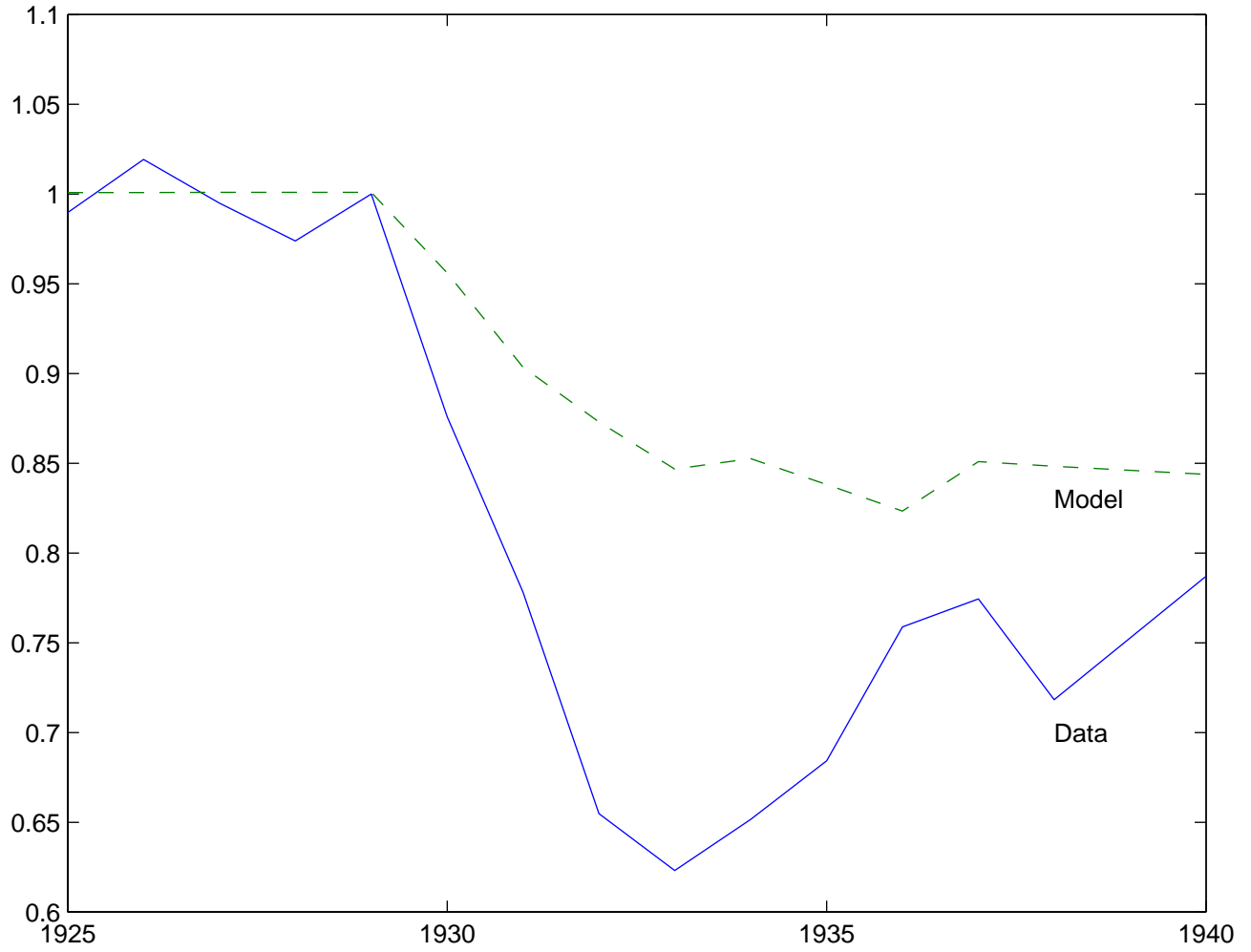


Figure 4b. Labor -- Actual and Simulated from Investment Model

