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8107 Macroeconomic Theory, Spring 2012, Mini 1

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Lecture 3. Theory of distributions with representative consumers

In this lecture we consider a dynamic economy in which there is heterogeneity in wealth endowments and in which the representative agent result applies. Our focus is on how aggregate dynamics affect the dynamics of the wealth and consumption distribution (although the wealth distribution and its dynamics do not affect aggregate dynamics). This is probably the simplest framework within which we can study the determination and the evolution of distributions, such as the wealth distribution, the income distribution or the consumption distribution. Key references that you should read are the papers by Chatterjee (JPUBE, 1994) and Caselli and Ventura (AER 2000) which are available on the class page. For a very recent application of this type of models see the recent paper by Glover et al. (NBER working paper 16924).

1 The economy

Demographics and preferences– The economy is inhabited by N types of infinitely lived agents, indexed by $i = 1, 2, ..., N$. Denote by μ^i the measure of agents i and normalize the total measure of agents to one, i.e $\sum_{i=1}^{N} \mu^i = 1$. Since the mass of agents is 1, from now on all aggregate variables can also be interpreted as per capita variables. Preferences are time separable, defined over streams of consumption, given by

$$U = \sum_{t=0}^{\infty} \beta^t u\left(c_t^i\right),$$

For the purpose of this example (see Chatterjee for more general cases) assume that $u(c) = \log(\bar{c} + c)$, with $\bar{c} + c \geq 0$ where we allow $\bar{c} \leq 0$ in order to be able to model a subsistence level for consumption. Notice that in this economy agents differ in their level of wealth and that there is uncertainty (neither idiosyncratic nor aggregate).

Household's problem– We first assume complete markets, so we can use the Arrow-Debreu formulation of the household problem (with the time-zero lifetime budget constraint). The maximization problem of household i can therefore be stated as (normalizing $p_0 = 1$)

$$\max_{\{c_t^i\}} \sum_{t=0}^{\infty} \beta^t u\left(c_t^i\right)$$

$$s.t.$$

$$\sum_{t=0}^{\infty} p_t c_t^i \leq a_0^i = s_0^i \sum_{t=0}^{\infty} p_t d_t \tag{1}$$

where p_t is the price of the consumption at time t relative to consumption at time 0, a_0^i is the initial wealth of agent i in terms of time 0 consumption and d_t represent dividend paid by a representative firm (see below). A different (and perhaps more appealing) assumption is that the household only trades a stock in each period. Let s_t^i be the share of the stock held by the household i in period t and let q_t be the price of the stock at time t. We can then rewrite the problem of the agent as

$$\begin{aligned}
&\max_{\{c_t^i, s_{it}\}} \sum_{t=0}^{\infty} \beta^t u\left(c_t^i\right) \\
&s.t. \\
&s_{it}(q_t + d_t) = c_{it} + s_{it+1} q_t \qquad \text{for every } t
\end{aligned} \tag{2}$$

In this deterministic environment the two problems have the same solution, in particular we can prove the following result. Let $\{c_{it}^{CM}\}$ be the solution to the complete markets household problem for a given process for p_t and d_t. Define the price of the stock at time t (in units of time t consumption) to be

$$q_t = \sum_{j=1}^{\infty} \frac{p_{t+j}}{p_t} d_{t+j}$$

Let $\{c_{it}^S\}$ be the solution to the stock trading household problem for a given process for q_t and d_t. Then $\{c_{it}^S\} = \{c_{it}^{CM}\}$. To show this we need to show that the first order conditions of the two problems are the same and that the budget constraints in the two problems are the same. To show the equivalence of the first order conditions note that in complete markets the first order conditions are

$$u'\left(c_t^i\right) = \beta \frac{p_t}{p_{t+1}} u'(c_{t+1}^i)$$

In the stock economy they are

$$q_t u'\left(c_t^i\right) = \beta u'(c_{t+1}^i)(q_{t+1} + d_{t+1}) \tag{3}$$

note now that

$$\begin{aligned}
q_{t+1} + d_{t+1} &= \sum_{j=2}^{\infty} \frac{p_{t+1+j}}{p_{t+1}} d_{t+j} + d_{t+1} \\
&= \frac{p_t}{p_{t+1}} \left[\sum_{j=2}^{\infty} \frac{p_{t+1+j}}{p_{t+1}} d_{t+j} + d_{t+1} \right] \frac{p_{t+1}}{p_t} \\
&= \frac{p_t}{p_{t+1}} \left[\sum_{j=2}^{\infty} \frac{p_{t+1+j}}{p_t} d_{t+j} + d_{t+1} \frac{p_{t+1}}{p_t} \right] \\
&= \frac{p_t}{p_{t+1}} \left[\sum_{j=1}^{\infty} \frac{p_{t+j}}{p_t} d_{t+j} \right] \\
&= \frac{p_t}{p_{t+1}} q_t
\end{aligned}$$

This result can be summarized as

$$p_{t+1}\left(q_{t+1} + d_{t+1}\right) = p_t q_t \tag{4}$$

which simply shows that the value of a stock today (after dividends are being paid) is equal to the value of the stock tomorrow plus tomorrw's dividends. Substituting (4) into (3) yields the result.

To show that the sequence of budget constraints in the stock economy is equivalent to the single complete markets economy, start from the first constraint in the stock economy

$$c_{i0} + s_{i1}q_0 = s_{i0}(q_0 + d_0) \tag{5}$$

and note that

$$(q_0 + d_0) = \sum_{t=0}^{\infty} p_t d_t$$

so that (5) can be rewritten as

$$c_{i0} + s_{i1}q_0 = s_{i0} \sum_{t=0}^{\infty} p_t d_t \tag{6}$$

now notice that using the budget constraint at time 1 we can write

$$s_{i1} = \frac{c_{i1}}{(q_1 + d_1)} + s_{i2}q_1 \tag{7}$$

substituting (7) into (6) yields

$$c_{i0} + \frac{c_{i1}q_0}{(q_1 + d_1)} + s_{i2}q_1q_0 = s_{i0} \sum_{t=0}^{\infty} p_t d_t \tag{8}$$

now remeber (from 4) that $(q_1 + d_1) = \frac{p_0}{p_1} q_0$ and the normalization $p_0 = 1$ then (8) can be written as

$$c_{i0} + c_{i1}p_1 + c_{i0} + s_{i2}q_1q_0 = s_{i0} \sum_{t=0}^{\infty} p_t d_t$$

doing repeated substitution yields

$$\sum_{t=0}^{\infty} p_t d_t = s_0^i \sum_{t=0}^{\infty} p_t d_t$$

which is the complete markets budget constraint.

Note also that, in general, we can define a_t^i the total wealth (including dividend payments) of agent i at time t (measured in units of time t consumption) as

$$a_t^i = s_t^i \sum_{j=t}^{\infty} \frac{p_j}{p_t} d_j = s_t^i (d_t + q_t), \tag{9}$$

where s_t^i is the share of the firm-value owned by consumer i at time t. Indeed, by summing both sides of (9) over i and exploiting the fact that $\sum_{i=1}^{N} \mu_i s_t^i = 1$ for every t, we obtain

$$\sum_{i=1}^{N} \mu_i s_t^i a_t^i = a_t = (d_t + q_t)$$

where $(d_t + q_t)$ is the total value of the representative firm (including current dividend).

Technology and firm's problem– Assume that there is a large number of firms and that each firm can operate an identical technology that allows to transform k units of consumption good today into $(1-\delta)k + f(k)$ units of consumption tomorrow with f strictly increasing, strictly concave and differentiable. Assume also that each firm starts out with the same amount of consumption good k_0.It can be shown that each firm will choose the same production plan.

> **Homework**
> Show that is indeed the case. What happens if each firm starts with a different k_0? Can you still construct a representative firm? if not what else do you need to keep track to describe the evolution of the production sector?

If that is the case we can focus on a representative firm which owns physical capital and makes the investment decision by solving the problem

$$\begin{aligned} a_0 = \max_{\{k_j\}} \sum_{j=0}^{\infty} p_j \left[f\left(k_j\right) + (1-\delta)\, k_j - k_{j+1} \right] & \\ = \max_{\{k_j\}} \sum_{j=0}^{\infty} p_j d_j & \\ d_j \equiv \left[f\left(k_j\right) + (1-\delta)\, k_j - k_{j+1} \right] & \\ k_0 \text{ given} & \end{aligned}$$

It is also easy to see that we can write the value of the firm at an arbitrary period (in units of time 0 consumption) as

$$p_t a_t = \max_{\{k_j\}} \sum_{j=t}^{\infty} p_j \left[f\left(k_j\right) + (1-\delta)\, k_j - k_{j+1} \right]$$

> **Homework**
> Assume that in this economy there is a constant mass $L = 1$ of workers, that the technology is $f(K, L) = L^{1-\alpha} K^{\alpha} = K^{\alpha}$ and that in each period the representative firm hires workers and pays dividends $d_t = \alpha k_t^{\alpha} + (1-\delta)k_t - k_{t+1}$ to its stockholders while it pays $(1-\alpha)k_t^{\alpha}$ to workers. Show that in this case the value of the firm $a_t = k_{t+1}$. Is this true also in the economy in which there are no workers but a fixed factor and so $d_t = k_t^{\alpha} + (1-\delta)k_t - k_{t+1}$ (i.e. stock holders receive also the remuneration to the fixed factor)? Explain why.

Solution to the household's problem From the FOC of the household problem, we have:

$$\beta^t u'\left(c_t^i\right) = \lambda^i p_t \quad \Rightarrow \quad \beta^t \left(\frac{1}{\bar{c} + c_t^i}\right) = \lambda^i p_t \quad \Rightarrow \quad c_t^i = \frac{\beta^t}{\lambda^i p_t} - \bar{c}, \tag{10}$$

where λ_i is the Lagrange multiplier on the budget constraint of household i. Substituting this FOC into the time-zero lifetime budget constraint (1), we can derive an expression for the multiplier λ^i:

$$\begin{aligned}
\sum_{t=0}^{\infty} p_t \left(\frac{\beta^t}{\lambda^i p_t} - \bar{c} \right) &= a_0^i \\
\frac{1}{\lambda^i (1-\beta)} - \bar{c} \sum_{t=0}^{\infty} p_t &= a_0^i \\
\left(\frac{1}{\lambda^i} \right) &= (1-\beta)\, a_0^i + (1-\beta)\, \bar{c} \sum_{t=0}^{\infty} p_t \qquad (11)
\end{aligned}$$

Let's now substitute the expression on the last line into equation (10) evaluated at time $t = 0$ (remember that $p_0 = 1$) in order to solve explicitly for c_0^i:

$$\begin{aligned}
c_0^i &= \left[(1-\beta)\, a_0^i + (1-\beta)\, \bar{c} \sum_{t=0}^{\infty} p_t \right] - \bar{c} \\
&= \bar{c} \left[(1-\beta) \sum_{t=0}^{\infty} p_t - 1 \right] + (1-\beta)\, a_0^i \qquad (12) \\
&= \Theta\left(p^0, \bar{c}\right) + (1-\beta)\, a_0^i,
\end{aligned}$$

where $\Theta\left(p^0, \bar{c}\right)$ denotes a function of the subsistence level and of the whole price sequence $p^0 = \{p_0, p_1, ..., p_t, ...\}$.This derivation can be easily generalized for every $t > 0$ (by using the Arrow-Debreu constraint for time t) so that

$$c_t^i = \Theta\left(p^t, \bar{c}\right) + (1-\beta)\, a_t^i, \qquad (13)$$

which shows that the optimal consumption choice at time t is an *affine function* of asset holdings at time t for each type i.

More in general, when period utility belongs to the families considered by Chatterjie, then preferences share a common property. They are *homothetic*, i.e. have linear Engel curves in wealth: any given change in wealth induces the same change in consumption, independently of the wealth level .[1]Even though we have only derived it for the log-case, it is easy to check that this representation of the consumption function holds also for the other two classes of preferences considered in the paper (power and exponential utility).

Representative agent result The first consequence of equation (13) is that to study the dynamics of aggregate variables (i.e. prices and quantities) in this model economy, we don't need to keep track of the distribution of wealth. From (13), we derive easily that aggregate consumption only depends on aggregate variables (prices and aggregate wealth), i.e.

$$c_t = \Theta\left(p^t, \bar{c}\right) + (1-\beta)\, a_t, \qquad (14)$$

where a_t can be clearly be expressed only as a function of the sequence of prices $\{p_t\}_{t=0}^{\infty}$ and aggregate capital stocks $\{k_t\}_{t=0}^{\infty}$. The competitive aggregate quantities and prices can therefore be recovered

[1]Technically, when $\bar{c} < 0$, preferences are quasi-homothetic because the Engel curves do not start at the origin, i.e. they are not linear but affine. However, linearity of the wealth-expansion path is not affected by the constant $\bar{c}$.

solving the equilibrium with a single consumer which is endowed with average wealth. Invoking the first welfare theorem those allocation can be solved solving the following standard single-agent planning problem:

$$\begin{aligned} & \max_{\{c_t\}} \sum_{t=0}^{\infty} \beta^t u(c_t) \\ & s.t. \\ & c_t + k_{t+1} \leq f(k_t) + (1-\delta) k_t \\ & k_0 \text{ given} \end{aligned} \tag{PP}$$

with standard first order condition

$$u'(c_t) = \beta u'(c_{t+1}) \left[f'(k_{t+1}) + (1-\delta) \right].$$

Note that, from (10) and the equation above, we can obtain equilibrium prices through the recursion

$$\frac{p_t}{p_{t+1}} = f'(k_{t+1}) + (1-\delta), \text{ where } p_0 \equiv 1, \quad k_0 \text{ given} \tag{15}$$

Steady-state– It is easy to show that the economy will converge to the steady-state values of capital stock satisfying $f'(k^*) = 1/\beta - (1-\delta)$. Note now that in steady-state the interest rate (i.e. the price of today's good relative to tomorrow's good), is equal to p_t/p_{t+1} which is equal to $1/\beta$ for all t, hence from the definition of $\Theta(p^t, \bar{c})$ in (12) we conclude that $c^i = (1-\beta) a^i$. In other words, in steady-state, the average propensity to save out of wealth is $\beta = \frac{r}{1+r}$ independently of wealth, for every type of household.

2 Equilibrium dynamics of the wealth distribution

The results described above imply that the dynamics of the aggregate variables are not affected by the evolution of the wealth distribution, but the inverse statement is not true: in general, the evolution of the wealth distribution across households (i.e. wealth inequality) depends on the dynamics of aggregate variables (prices and quantities).

To see this, note that from the budget constraint of agent i at time t (using 9 and 4)

$$\begin{aligned} c_t^i + s_{t+1}^i q_t &= s_t^i (d_t + q_t) \quad \Rightarrow \quad p_t c_t^i + s_{t+1}^i (d_{t+1} + q_{t+1}) p_{t+1} = s_t^i (d_t + q_t) p_t \Rightarrow \\ p_t c_t^i + p_{t+1} a_{t+1}^i &= p_t a_t^i \qquad (16) \\ \frac{p_t c_t^i}{p_t a_t^i} + \frac{p_{t+1} a_{t+1}^i}{p_t a_t^i} &= 1 \quad \Rightarrow \quad \frac{p_{t+1} a_{t+1}^i}{p_t a_t^i} = \left(1 - \frac{c_t^i}{a_t^i}\right), \qquad (17) \end{aligned}$$

which expresses the growth rate of wealth for type i as a function of her consumption-wealth ratio. Note now that, from equations (13) and (14),

$$\frac{c_t^i}{a_t^i} = \frac{\Theta(p^t, \bar{c})}{a_t^i} + (1-\beta), \quad \text{and} \quad \frac{C_t}{A_t} = \frac{\Theta(p^t, \bar{c})}{A_t} + (1-\beta)$$

Thus, putting together this last line and (17) tells me that whether wealth of individual i grows faster or slower than the average depends on whether its consumption to wealth ratio is lower or higher than the average, which in turn depends on the sign of the constant $\Theta\left(p^t, \bar{c}\right)$ and on her relative position in the distribution $\left(a_t^i - A_t\right)$

$$\frac{a_{t+1}^i}{a_t^i} \gtrless \frac{A_{t+1}}{A_t} \quad \Leftrightarrow \quad \frac{c_t^i}{a_t^i} \lessgtr \frac{C_t}{A_t} \quad \Leftrightarrow \Theta\left(p^t, \bar{c}\right)\left(a_t^i - A_t\right) \gtrless 0 \tag{18}$$

In other words,

$$\Theta\left(p^t, \bar{c}\right)\left(a_t^i - A_t\right) \gtrless 0 \quad \Leftrightarrow \quad \frac{s_{t+1}^i}{s_t^i} \gtrless 1$$

This result leads easily to the following:

Result 1: The wealth distribution remains unchanged if either of the two conditions are satisfied:

i) The economy starts with capital stock equal to its steady state level

ii) $\bar{c} = 0$

In presence of a subsistence level and with the economy starting out of the steady state things change. For example, if $\Theta > 0$ and $a_t^i > A_t$, then consumer i wealth share will grow over time, hence the distribution will become more unequal. We now determine the sign of Θ, through:

Lemma 1.1 (Chatterjee, 1994): *The common constant term of the consumption function* $\Theta\left(p^t, \bar{c}\right)$ *is greater, equal or less than zero if and only if* $\bar{c}\left(k_t - k^*\right)$ *is greater, equal or less than zero.*

Proof: *Suppose the economy grows towards the steady-state, i.e.* $k_t < k^*$*. From equation* (15), *the sequence* $\{f'(k_t)\}$ *is decreasing and the sequence* $\{p_{t+1}/p_t\}$ *will be increasing towards* β*. Therefore,* $p_{\tau+1}/p_\tau \leq \beta$ *for all* $\tau \geq t$ *where the strict inequality holds at least for some* t*. It follows that*

$$p_\tau/p_t = \left(p_\tau/p_{\tau-1}\right)\left(p_{\tau-1}/p_{\tau-2}\right)...\left(p_{t+2}/p_{t+1}\right)\left(p_{t+1}/p_t\right) < \beta^{\tau-t}.$$

From the definition of $\Theta\left(p^t, \bar{c}\right)$ *in* (12), *use the above equation to obtain*

$$\Theta\left(p^t, \bar{c}\right) = \bar{c}\left[(1-\beta)\sum_{\tau=t}^{\infty}\left(\frac{p_\tau}{p_t}\right) - 1\right] > \bar{c}\left[(1-\beta)\sum_{\tau=t}^{\infty}\beta^{\tau-t} - 1\right] = 0$$

where the first inequality follows from $\bar{c} < 0$. **QED**

The consequences for the evolution of the wealth distribution in an economy growing towards the steady-state are easy to determine, at this point. In the presence of a subsistence level ($\bar{c} < 0$), $\Theta > 0$ in a growing economy. $\Theta > 0$ implies that the average propensity to consume (save) declines (increases) with wealth. In fact, from equation (18), it is clear that agents with wealth above average will increase their wealth even more relatively to the average. In other words:

Result 2: If $\bar{c} < 0$*: (i) the wealth distribution becomes more unequal as the economy grows towards the steady-state, as rich agents accumulate more than poor agents along the transition path, and (ii) there is no change in the ranking of households in the wealth distribution, i.e., initial conditions in ranking persist forever.*

Intuition A brief discussion is in order on why we obtain this result. The key piece here is that complete markets implies Pareto efficiency, which implies constant ratios of marginal utilities of consumption at an given date for any two agents. What does equalization of marginal utilities implies for consumption? This clearly depends on preferences. If preferences are simple logarithmic ($\bar{c} = 0$) constant marginal utility ratio implies constant consumption ratios. Note that in the log case the consumption function is $c_{it} = (1 - \beta)a_{it}$. Now if $a_{it} = s_{i0}a_t$,that is if wealth ratios between any two agents are constant then consumption ratios between any two agents will also be constant; in other words a constant wealth distribution decentralize the efficient allocation. If $\bar{c} \neq 0$ then constant ratios of marginal utilities simply does not imply constant ratios of consumption. To see this consider the simple case of two consumers, i and j. In a complete markets equilibrium the ratio of their marginal utilities is constant (why?) and equal to κ

$$\frac{u'(c_{it})}{u'(c_{jt})} = \frac{c_{jt} + \bar{c}}{c_{it} + \bar{c}} = \kappa$$

dividing both numerator and denominator by c_{jt} you get

$$\frac{1 + \bar{c}/c_{jt}}{c_{it}/c_{jt} + \bar{c}/c_{jt}} = \kappa$$

and note that (besides the degenerate case in which $\kappa = 1$ and $c_{it}/c_{jt} = 1$, which corresponds to the case of no heterogeneity) the consumption ratio c_{it}/c_{jt} cannot be constant, if c_{jt} grows over time. In other words with these preferences the aggregate level of resources matters for distribution. Suppose for example $\bar{c} < 0$ and that the economy grows. At low level of resources marginal utility of the poor agent (which is closer to its subsistence level) is much higher than the one of the rich agent, while at high level of resources the marginal utilities are much similar (because the subsistence level is less important for both). It is therefore efficient for the poor agent to consume (relatively) more early and (relatively) less late, so that it is efficient to have a growing path of consumption inequality, which is implemented with a path of growing wealth inequality.

Robustness– We now discuss how robust this result is to some of the key assumptions made so far in the analysis: 1) all agents have same discount factor β, 2) markets are complete, 3) absence of leisure and heterogeneity in efficiency units of labor.

- Suppose agents have different discount factors and suppose that $\bar{c} = 0$ to simplify the analysis. Then, from (13)

$$c_t^i = \left(1 - \beta^i\right) a_t^i,$$

  therefore the average propensity to save out of wealth is higher the more patient is the individual and from (18), wealth grows faster for the more patient individuals. In this case, to characterize aggregate dynamics of wealth one needs to know the entire distribution of wealth. Notice also that in the limit, in steady-state, the most patient type holds all the wealth, and the distribution is degenerate.

- In absence of markets (autarky), every consumer has access to her own technology, but there is no trade. Each agent will solve a standard planning problem with different initial conditions K_0^i. It is easy to see that, independently of the initial conditions, each agent will converge to the same capital stock K^*, hence in the long-run the distribution of wealth is perfectly equal.

Interestingly, we conclude that a less developed financial market induces,in the long-run, less consumption and wealth inequality. Keep in mind though that in terms of welfare, in an ex-ante sense, all consumers are better off in complete markets. Ex post a consumer which starts poor, in the long run consumes more in incomplete markets, but this extra consumption is not free as it is a result of low initial consumption and higher investment by this consumer. This is an important point as it suggest that mobility is not necessarily a desirable feature of allocations: the incomplete markets economy display social mobility, while the complete markets display no social mobility, yet (ex-ante) welfare is higher in complete markets.

Also, there is an important parallel with the convergence literature in growth theory: just think of consumers as countries. If every country has access to the same world technology and capital is perfectly mobile across countries, then the neoclassical growth model does *not*predict convergence anymore. More precisely, $f'\left(k^i\right)$ would be equalized across countries, hence countries would have the same capital stock and produce the same output. Thus, there would be convergence in GDP, but not in GNP.

- When preferences are also a function of hours worked h, and households differ by their (fixed) endowment of efficiency units and by their holdings of shares of the representative firm, aggregation can also occur, under certain restrictions on preferences, for example when

$$u\left(c,h\right)=\frac{\left(c^{\alpha}\left(1-h\right)^{1-\alpha}\right)^{1-\gamma}}{1-\gamma}. \tag{19}$$

Homework: Show that this indeed the case.

3 Indeterminacy of the wealth distribution in steady-state

One very important implication of the aggregation results is that in steady-state the wealth distribution is indeterminate: this means that an environment with complete markets does not offer a theory of the initial (or final) wealth distribution, but only of its evolution. Suppose again that $\bar{c}=0$ to simplify the analysis. In this case the set of equations characterizing the steady-state is

$$\begin{aligned}
c^i &= \left(1-\beta\right)a^i,\ i=1,2,...,N \\
a^i &= s^i\frac{1}{1-\beta}\left[f\left(K^*\right)-\delta K^*\right],\ i=1,2,...,N \\
f'\left(K^*\right) &= 1/\beta-\left(1-\delta\right), \\
\sum_{i=1}^{N}\mu^i s^i &= 1,
\end{aligned}$$

We therefore have $(2N+2)$ equations and $(3N+1)$ unknowns $\left(\left\{c^i,a^i,s^i\right\}_{i=1}^N,K^*\right)$. In other words, the multiplicity of the steady-state wealth distributions is of order $N-1$.[2]

[2]This means that, if $N=1$ (representative agent), the steady-state is unique. If $N=2$, there is a continuum of steady-states of dimension 1, and so on.

However, suppose we start from a given wealth distribution at time 0 when the economy has not yet reached its steady-state, then the dynamics of the model are uniquely determined by results above and the final steady-state distribution is determined as well. So, let's restate this as:

Result 3: In the steady-state of the neoclassical growth model with N agents heterogeneous in initial endowments and homothetic preferences, there is a continuum of steady-state wealth distributions, with dimension $(N-1)$. However, given an initial wealth distribution $\left\{s_0^i\right\}_{i=1}^N$ at $t=0$, the equilibrium wealth distribution $\left\{s_t^i\right\}_{i=1}^N$ in every period t is uniquely determined, and so is the final steady-state distribution.

Note that in incomplete markets this is not the case as the steady state wealth distribution is uniquely determined (in this simple example it would be the perfectly egalitarian distribution) for any possible initial condition. This is a very general point which is related to the mobility argument above: in complete markets households insure against unforseen consequences (good or bad) and hence initial differences persist forever. In incomplete markets luck matters, so in the long run the effects of initial condition vanish and the steady state distribution is only shaped by luck.

Finally, in terms of language, this whole section shows that it important to distinguish "steady-state" from "equilibrium path". In this economy, the equilibrium path is always unique (given initial conditions), but the complete markets steady-state is not. In the incomplete markets economy both the equilibrium path and the steady state are uniquely determined (given initial conditions).