



## Macroeconomic Theory (8107)

### Fall 2009, Mini 2

#### Problem set 4

**Question 1.** Consider a closed economy inhabited by a continuum of infinitely lived agents (each agent is indexed by  $i$ ) with preferences

$$E \sum_{t=0}^{\infty} \beta^t \left( \log(c_{it}) - \frac{l_{it}^{\phi}}{\phi} \right)$$
$$1 > \beta > 0, \phi > 1$$

where  $c_{it}$  is consumption (of a single good) and  $l_{it}$  is labor. Agents have no initial wealth, and in each period they are endowed with labor productivity  $e_{it}$  given by

$$e_{it} = \exp(\varepsilon_{it} + A_i)$$

where

$$A_i \rightarrow N\left(\frac{-\sigma_A^2}{2}, \sigma_A^2\right) \text{ i.i.d. across agents, constant through time}$$
$$\varepsilon_{it} \rightarrow N\left(\frac{-\sigma_{\varepsilon}^2}{2}, \sigma_{\varepsilon}^2\right) \text{ i.i.d. across agents and time}$$

Agents can trade a full set of Arrow securities contingent on the realization of their  $\varepsilon$  and can sell their effective labor  $e_{it}l_{it}$  to competitive firms who use a constant returns to scale technology that transforms one unit of effective labor into one unit of consumption good. Let  $w$  be the price of one unit of effective labor in terms of the consumption good. There is no capital nor aggregate uncertainty in the economy. Assume throughout that appropriate law of large numbers apply, so that for each period and for each possible value of the permanent shock  $A$ , the measure of agents experiencing temporary shocks in a given set is constant.

1. Denote with  $p_{\varepsilon'}(A, \varepsilon)$  and  $b_{\varepsilon'}(A, \varepsilon)$  the price and the quantity purchased (by an agent experiencing shocks  $A, \varepsilon$ ) of an Arrow security paying off 1 unit of

consumption tomorrow if the agent tomorrow experiences shock  $\varepsilon'$ , and with  $c(A, \varepsilon), l(A, \varepsilon)$  consumption and labor supply of the same agent. Define a competitive equilibrium for this economy

2. Show that in equilibrium

$$p_{\varepsilon'}(A, \varepsilon) = \beta\pi(\varepsilon')$$

where  $\pi(\varepsilon')$  is the pdf of  $\varepsilon'$ ,

$$c(A, \varepsilon) = c(A)$$

i.e. consumption is independent from the temporary shock  $\varepsilon$ , and

$$b_{\varepsilon'}(A, \varepsilon) = c(A) - wl(A, \varepsilon') \exp(\varepsilon' + A)$$

3. Solve for the risk free rate and for the wage per unit of effective labor in this economy
4. Solve for  $c(A)$  and for  $l(A, \varepsilon)$
5. Show that aggregate output and ex-ante welfare (i.e. welfare before you know your  $A$  and the sequence of  $\varepsilon$ ) in this economy are increasing in  $\sigma_{\varepsilon}^2$ . Explain why
6. Show that aggregate output is constant in  $\sigma_A^2$  and ex ante welfare is decreasing in  $\sigma_A^2$ . Explain why
7. Solve for the difference between labor income inequality and consumption inequality (measure inequality as the standard deviations of the log). What are the key parameters that determine this difference? Explain why

**Question 2.** Consider an economy with a continuum (measure 1) of ex-ante identical consumers, each living for two periods. The consumers have utility given by

$$\log(c_1) + \beta \log(c_2)$$

where  $c_1$  and  $c_2$  are period 1 and period 2 consumption and  $\beta$  is the discount factor. In period 1, each agent is endowed with  $y$  units of output which can be either consumed,  $c_1$ , or invested,  $k$ . In period two, consumers receive income from the capital they saved in period 1 and from inelastically supplying their labor endowment to the market. The labor endowment of any given individual is random and it is independent across agents. Period-2 labor endowments can be either  $1 - \varepsilon$  or  $1 + \varepsilon$ , with  $0 < \varepsilon < 1$ ; the probability that any agent's labor endowment is  $1 + \varepsilon$  is  $1/2$ . Due to the independence of shocks across consumers, a law of large numbers operates so that also the fraction of agents with labor endowment in period 2 equal to  $1 + \varepsilon$  is  $1/2$ . That is, there is no uncertainty about the period-2 aggregate labor endowment: the supply of labor is constant at 1. In the second period, output is produced by perfectly competitive firms which operate a standard Cobb-Douglas production function: they sell the output to consumers and rent the capital and the labor services from the same consumers at rates  $r$  and  $w$ , respectively.

1. Write down the consumers problem and define a competitive equilibrium for this economy
2. For the case  $\varepsilon = 0$  (no individual uncertainty) analytically solve for the competitive equilibrium. Argue that the equilibrium is efficient.
3. Show that the equilibrium interest rate is a decreasing function of  $\varepsilon$ .
4. For the case  $\varepsilon > 0$  define the equilibrium risk faced by consumers as the ratio between equilibrium consumption in the high endowment state and equilibrium consumption in the low endowment state. Show that a) equilibrium risk is *not* affected by the equilibrium level of  $k$  and b) agents perceive that their individual risk is affected by their choice of  $k$ .
5. For the case  $\varepsilon > 0$  show that the equilibrium capital is not efficient in the sense that ex ante welfare of agents can be improved simply by changing the level of  $k$ ?

For more on this issue you can take a look at the paper: "Constrained efficiency in the neoclassical growth model with uninsurable idiosyncratic shocks, by Davila, Hong, Krusell and Rios-Rull.

**Question 3.** Consider an economy with a continuum (measure 1) of ex-ante identical consumers each having efficiency units of labor  $\varepsilon$  which is i.i.d. over time and across agents, has non negative support and mean  $\mu$ . Consumers can trade a non contingent bond  $a$  but cannot borrow ( $a \geq 0$ ). Suppose you already know that their optimal decision rules for assets have this form

$$a' = \max((1 + r)a + \gamma w \varepsilon, 0)$$

where  $0 < \gamma < 1$  is a constant and  $r$  and  $w$  are equilibrium interest and wage rates. There are also competitive firms which hire labor and capital at prices  $w$  and  $r$  and operate a standard production function  $AK^\alpha L^{1-\alpha} + (1 - \delta)K$  where  $A$  is productivity and  $1 > \delta > 0$  is the depreciation of capital

1. Show that a stationary distribution of consumers over assets and shocks exist only for  $r < 0$ .
2. For all  $r < 0$  solve and plot the stationary supply of assets i.e.  $E(a)$  and for demand for capital by the firms i.e.  $K(r)$
3. Solve for equilibrium interest rate and wage rate.
4. Discuss what happens to long run equilibrium GDP, interest and wage rate if consumer want to save more ( $\gamma$  increases) or if productivity of the firms decline ( $A$  goes down) .