University of Minnesota

8107 Macroeconomic Theory, Fall 2009, Mini 2

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Lecture 4. Aggregation with non homothetic preferences and skill heterogeneity

In this class we will consider a simplified version of the model described by Maliar and Maliar (2003) who prove a generalized version of the representative agent result in the case in which there is heterogeneity in skills and in which preferences are not homothetic. The economy is inhabited by a continuum of measure 1 of infinitely lived agents, indexed by i. There is heterogeneity in initial wealth endowments (denoted by k_i) and agents are subject to i.i.d productivity shocks to skills but there is no aggregate uncertainty. Let z_i be the skill shock (i.e. shock to its labor endowment) of agent i. Preferences are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^i, l_t^i)$$

$$u(c, 1-l) = \frac{c^{1-\sigma}}{1-\sigma} + A\frac{(1-l)^{1-\gamma}}{1-\gamma}$$

note that these preferences are not homothetic, meaning that wealth expansion paths are not linear, unless $\sigma = \gamma$ so in general the representative agent result will not hold. We assume complete financial markets, i.e. agents can trade a full set of state-contingent claims (denoted by $b(z_{t+1}^i)$) at prices $q(z_{t+1}^i)$ that allow them to completely insure against their their own skill shocks. Note that the assumption of no aggregate uncertainty guarantees that these person specific assets are sufficient for all agents to fully insure, i.e. there is no need on assets that are contingent on the aggregate state of the economy. There are also competitive firms who hire effective labor h at rate w and rent capital k at rate r to produce output using a standard CRS technology given by

$$y = k^{\alpha} h^{1-\alpha}$$

The agent's problem can be written as

$$\max_{c_t^i, k_{t+1}^i, l_t^i, b(z_t^i)} E \sum_{t=0}^{\infty} \beta^t u(c_t^i, 1-l_t^i)$$

$$s.t.$$

$$c_t^i + k_{t+1}^i + \int_{z_{t+1}^i} q(z_{t+1}^i) b(z_{t+1}^i) = (1-\delta+r_t)k_t^i + w_t z_t^i l_t^i + b(z_t^i)$$

$$b(z_0^i), k_0^i \ given$$

Because of the complete markets assumption and lack of distortions equilibrium allocations in this economy can be characterized using a planning problem which attaches weight λ_i to every agent. We first are going to define the following aggregate variables:

$$c_t = \int_i c_t^i, \qquad k_t = \int_i k_t^i, \qquad h_t = \int_i z_t^i l_t^i$$

and then write the planning problem in two steps. Step 1 is a static problem and is given by

$$U(c_t, 1-h_t, \{z_t^i, \lambda^i\}) \equiv \max_{c_t^i, l_t^i} \int_i \lambda_i u(c_t^i, 1-l_t^i) \tag{1}$$
$$s.t$$
$$c_t = \int_i c_t^i, \qquad h_t = \int_i z_t^i l_t^i$$

while step 2 is dynamic and is given by

$$\max_{c_t, h_t, k_{t+1}} E \sum_{t=0}^{\infty} \beta^t U(c_t, 1-h_t, \{z_t^i, \lambda^i\})$$
$$s.t.$$
$$c_t + k_{t+1} = k_t^{\alpha} h_t^{1-\alpha} + (1-\delta)k_t$$
$$k_0 \text{ given}$$

Notice that here U is not (necessarily) the same as u of the individual agent. U represent the indirect utility of the planner of having available today aggregate consumption c_t and aggregate labor input h_t to distribute across agents. Notice that the representative agent result amounts to finding a form for U that i) is known, ii) does not depend (or depends in a simple fashion) on individual level variables i.e. $\{z_t^i, \lambda^i\}$. In order to find the form for U we write the first order conditions of step 1,

$$\lambda_i \left(c_t^i\right)^{-\sigma} = \theta_{1t}$$
$$\lambda_i A(1-l_t^i)^{-\gamma} = \theta_{2t} z_{it}$$

or

$$c_t^i = \left(\frac{\lambda_i}{\theta_{1t}}\right)^{\frac{1}{\sigma}} \tag{2}$$
$$(1-l_t^i) z_t^i = \left(\frac{\lambda_i A}{\theta_{2t}}\right)^{\frac{1}{\gamma}} \left(z_t^i\right)^{\frac{\gamma-1}{\gamma}} \tag{3}$$

where θ_{1t} and θ_{2t} are the lagrange multipliers on the two constraints in step 1. Next we integrate (2) and (3) across individuals (remember that $\int z_t^i = 1$) to get

$$c_t = \left(\frac{1}{\theta_{1t}}\right)^{\frac{1}{\sigma}} \int_i \lambda_i^{\frac{1}{\sigma}} \tag{4}$$
$$(1 - l_t z_t) = 1 - h_t = \left(\frac{A}{\theta_{2t}}\right)^{\frac{1}{\gamma}} \int_i \lambda_i^{\frac{1}{\gamma}} \left(z_t^i\right)^{\frac{\gamma-1}{\gamma}} \tag{5}$$

finally dividing c_t^i by c_t and $(1-l_t^i)z_t^i$ by $1-h_t$ we can get

$$c_t^i = \frac{\lambda_i^{\frac{1}{\sigma}}}{\int_i \lambda_i^{\frac{1}{\sigma}}} c_t$$
$$(1-l_t^i) = \frac{\lambda_i^{\frac{1}{\gamma}} \left(z_t^i\right)^{-\frac{1}{\gamma}}}{\int_i \lambda_i^{\frac{1}{\gamma}} \left(z_t^i\right)^{\frac{\gamma-1}{\gamma}}} (1-h_t) \tag{6}$$

that states that at an efficient allocation individual consumption is a fixed fraction of aggregate consumption (as we have seen in the example in the previous lecture) and individual leasure depends positively on the weight (i.e. richer agents will enjoy more leisure) but negatively on the skill (it is efficient for more skilled agents to enjoy less leisure). The final step to prove the representative agent result is to substitute expressions for c_t^i and $(1-l_t^i)$ into (1), yielding

$$\begin{aligned}
U(c_t, 1-h_t, \{z_t^i, \lambda^i\}) &= \int_i \lambda_i u(c_t^i, 1-l_t^i) \\
&= \frac{c_t^{1-\sigma}}{1-\sigma} \frac{\int_i \lambda_i \lambda_i^{\frac{1-\sigma}{\sigma}}}{\left(\int_i \lambda_i^{\frac{1}{\sigma}}\right)^{1-\sigma}} + A\frac{(1-h_t)^{1-\gamma}}{1-\gamma} \frac{\int_i \lambda_i \lambda_i^{\frac{1-\gamma}{\gamma}} \left(z_t^i\right)^{-\frac{1-\gamma}{\gamma}}}{\left(\int_i \lambda_i^{\frac{1}{\gamma}} \left(z_t^i\right)^{\frac{\gamma-1}{\gamma}}\right)^{1-\gamma}} \\
&= \frac{c_t^{1-\sigma}}{1-\sigma} \left(\int_i \lambda_i^{\frac{1}{\sigma}}\right)^{\sigma} + A\frac{(1-h_t)^{1-\gamma}}{1-\gamma} \left(\int_i \lambda_i^{\frac{1}{\gamma}} \left(z_t^i\right)^{\frac{\gamma-1}{\gamma}}\right)^{\gamma}
\end{aligned}$$

since the term $\left(\int_i \lambda_i^{\frac{1}{\sigma}}\right)^{\sigma}$ is constant we can multiply utility by it to get

$$\begin{aligned}
U(c_t, 1-h_t, \{z_t^i, \lambda^i\}) &= \frac{c_t^{1-\sigma}}{1-\sigma} + AX_t \frac{(1-h_t)^{1-\gamma}}{1-\gamma} \\
X_t &= \frac{\left(\int_i \lambda_i^{\frac{1}{\gamma}} \left(z_t^i\right)^{\frac{\gamma-1}{\gamma}}\right)^{\gamma}}{\left(\int_i \lambda_i^{\frac{1}{\sigma}}\right)^{\sigma}}
\end{aligned}$$

notice that we have found sort of RA result in the sense that we can determine the equilibrium path for aggregate variables c_t h_t and k_t using "almost" a single agent problem. Notice though that the preferences of the representative agent are different from preferences of individual agents along two important dimensions:

i) The utility of representative agent is a function of effective labor and not actual hours worked so the solution of the representative agent will not give you directly the solution for aggregate hours. Notice though that by integrating (6) it is easy to derive aggregate hours worked in the economy.

ii) The utility of the representative agent has a taste shifter X_t which in principle depends on the distribution of skills in the economy. For example assume that

$$\begin{aligned}
z_t^i &= 1 + \frac{\varepsilon_{it}}{\xi_t} \\
\varepsilon_{it} &= N(0, \sigma_\varepsilon) \\
\xi_t &= \rho \xi_{t-1} + \eta_t
\end{aligned}$$

i.e. the distribution of skills gets more dispersed when the variable ξ_t moves around, then you can see that X_t will be moving around with ξ_t, acting like a preference shock, even if the underlying structural preferences are not affected by any shock. One attractive feature of the preference shifter is that its dynamics do not depend on control variables so they can be determined easily. Notice that if the utility function is homothetic ($\gamma = \sigma$) and no skill heterogeneity is present (i.e. $z_i^t = 1$) then the standard representative agent result applies i.e. preferences of the representative agent are the same as preferences of each individual agent and initial distribution of wealth (summarized by the

λ_i) does not matter for aggregates. Finally note that if preferences are logarithmic in labor ($\gamma = 1$) skill heterogeity does not matter for aggregates: the reason is that the skill term enters linearly in equation (3) and thus when the first order condition are intergrated across agents only average skill matter.