University of Minnesota
8107 Macroeconomic Theory, Fall 2009, Mini 2
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Lecture 1. Aggregation

1 Introduction

Probably so far in the macro sequence you have dealt directly with representative consumers and representative firms, meaning that you have solved some maximization problem of a fictitious single agent or single firm, assuming that this agent or firm was "representative" of the whole economy. What does representative exactly mean? Obviously it does not mean that knowing some variables for the average agent (for example its consumption, its wealth or its investment) tells you everything about all agents in the economy. Knowing, for example, that average household wealth in 2006 in the US is $100000 does not tell you how many households there are who have 0 wealth. Representative here means that knowing the variables of the representative agent is enough to predict some aggregate variables (for example prices) and/or to predict the evolution of aggregate variables in the future. To continue with the previous example if knowing that average household wealth in 2006 in the US is $100000 is enough to know average asset prices in 2006 and to predict average household wealth in 2007, then, for some questions, we do not need to worry about keeping track of the entire wealth distribution. In this lecture we will discuss in detail when and why it is enough to keep track only of averages, both in static and dynamic frameworks.

2 Aggregate demand in partial equilibrium static framework

We start with an important result that is discussed in Mas-Colell et al. section 4.B which gives conditions under which we can ignore the distribution of wealth across consumers in order to know aggregate demand. Consider a static economy with I consumers and L goods and assume that each consumer has wealth w_i and standard preferences (potentially different for every consumer) over consumption bundles. Solving a standard maximization problem yields (vector valued) demand functions for each consumer $x_i(p, w_i)$ and aggregate demand

$$x(p, w_1, ..w_I) = \sum_i x_i(p, w_i) \tag{1}$$

where p is the vector of prices of each good. Equation (1) tells us that in order to know aggregate demand (and thus, for example in a general equilibrium context, prices) one needs to know the entire distribution of wealth. We now ask when and if we can write aggregate demand as

$$X(p, \sum_i w_i) = \sum_i x_i(p, w_i), \qquad \text{for every } p, (w_{1,..}w_I) \tag{2}$$

In order for (2) to hold it must be that for every change in the wealth distribution dw satisfying $\sum_i dw_i = 0$, for every p and for every $(w_{1,..}w_I)$

$$\sum_i \frac{\partial x_{li}(p, w_i)}{\partial w_i} dw_i = 0 \qquad \text{for every } l$$

but this can be true if and only if

$$\frac{\partial x_{li}(p, w_i)}{\partial w_i} = \frac{\partial x_{lj}(p, w_j)}{\partial w_j}, \qquad \text{for every } l, p, (i, j), (w_{1,..}w_I) \tag{3}$$

which means that any two individuals who face the same increment in wealth change the demand of any given good in exactly the same (across individuals) fashion. In words this means that if I give an extra dollar to each consumer, each consumer will exactly spend the same additional amount on, say, cherries. Graphically it means that wealth expansion paths for every consumer have to be linear in each good with the same (across consumers) slope (see Mas-Colell figure 4.B.1). It is easy to see that if this is the case in order to compute aggregate demand it is sufficient to know total wealth and it is not necessary to know how wealth is distributed. Proposition 4.B. 1 in Mas-Colell et al. gives necessary and sufficient condition for equation 3 to hold and obviously the conditions have to do with preferences. In particular it must be that the indirect utility function of each agent $V_i(p, w_i)$ must have the following form (also called the Gorman form)

$$V_i(p, w) = a_i(p) + b(p)w_i$$

How restrictive is this condition?

It is obviously pretty restrictive and it is easy to think of many different preferences in which this condition is not satisfied. But there are two interesting cases in which it is satisfied. One is the case in which all consumers have identical and homothethic preferences, where identical guarantees that $b(p)$ does not depend on i and homothetic guarantees that $b(p)$ does not depend on w.Another case is when preferences of all agents are quasilinear in the same good (for more on this see question 3 in the problem set). In the next section we will discuss under which condition the representative agent result applies for dynamic economies, in general equilibrium.

3 The representative agent result in a benchmark dynamic economy

The objective of this section is to provide an example in which aggregate variables (in this case prices) at each point in time do not depend on the distribution of resources among agents of the economy at the moment in time but only on average resources available at that point in time. So in other words it tells us under which conditions one can explain aggregate prices (such us, for example, interest rates or stock prices) using only aggregate variables, such as aggregate consumption or GDP.

Consider an economy populated by N individuals, indexed by $i \in I = \{1, 2, \ldots N\}$. Each individual is infinitely lived. In each period there is one nonstorable consumption good. Each individual household has a stochastic endowment process $\{y_t^i\}$ of this consumption good. Let s_t and $s^t = (s_0, \ldots s_t)$ the event and event history of this economy, respectively. and $\pi_t(s^t)$ denote the objective probabilities of event histories. Agents subjective probability beliefs are assumed to coincide with these objective probabilities. Note that there is no need of additional assumptions about the stochastic process; in particular the individual processes need not be Markov processes nor to be independent across agents.

Assume that $y_t^i \in Y$, a finite-dimensional set of cardinality M and define the aggregate state simply as $s_t = (y_t^1, \ldots, y_t^N)$.Also take the event s_0 as given .

A consumption allocation $\{(c_t^i(s^t))_{i \in I}\}_{t=0, s^t \in S^t}^T$ maps aggregate event histories s^t into consumption of agents $i \in I$ at time t. Preferences are identical across agents, additively time-separable and that agents discount the future at common subjective time discount factor $\beta \in (0, 1)$, so that the utility function takes the form

$$u^i(c^i) = \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi_t(s^t) U^i(c_t^i(s^t)) \tag{4}$$

Definition 1 *A consumption allocation $\{(c_t^i(s^t))_{i \in I}\}_{t=0, s^t \in S^t}^T$ is feasible if*

$$c_t^i(s^t) \geq 0 \text{ for all } i, t, s^t \tag{5}$$

$$\sum_{i=1}^{N} c_t^i(s^t) = \sum_{i=1}^{N} y_t^i(s^t) \text{ for all } t, s^t \tag{6}$$

A consumption allocation is Pareto efficient if it is feasible and there is no other feasible consumption allocation $\{(\hat{c}_t^i(s^t))_{i \in I}\}_{t=0, s^t \in S^t}^T$ such that

$$u^i(\hat{c}^i) \geq u^i(c^i) \text{ for all } i \in I \tag{7}$$

$$u^i(\hat{c}^i) > u^i(c^i) \text{ for some } i \in I \tag{8}$$

4 Complete Markets and the Representative Agent

The key assumption that will allow as to get the representative agent result is the one of complete markets. With complete markets we suppose that there is a complete set of contingent claims that are traded at time 0, before any uncertainty has been revealed. The individual Arrow Debreu budget constraints take the form

$$\sum_{t=0}^{\infty} \sum_{s^t \in S^t} p_t(s^t) c_t^i(s^t) \leq \sum_{t=0}^{\infty} \sum_{s^t \in S^t} p_t(s^t) y_t^i(s^t) \tag{9}$$

where $p(s^t)$ is the period 0 price of one unit of period t consumption, delivered if event history s^t has realized.

Arrow-Debreu Equilibrium

Definition 2 *An Arrow Debreu competitive equilibrium consists of allocations* $\{(c_t^i(s^t))_{i\in I}\}_{t=0,s^t\in S^t}^T$ *and prices* $\{p(s^t)\}_{t=0,s^t\in S^t}^T$ *such that*

1. *Given* $\{p(s^t)\}_{t=0,s^t\in S^t}^T$, *for each* $i \in I$, $\{c_t^i(s^t)\}_{t=0,s^t\in S^t}^T$ *maximizes* (4) *subject to* (5) *and* (9)
2. $\{(c_t^i(s^t))_{i\in I}\}_{t=0,s^t\in S^t}^T$ *satisfies* (6) *for all* t, s^t.

Now let us make the following assumption

Assumption 1: *The period utility functions* U^i *are twice continuously differentiable, strictly increasing, strictly concave in its first argument and satisfy the Inada conditions*

$$\lim_{c\to 0} U_c^i(c) = \infty \tag{10}$$

$$\lim_{c\to\infty} U_c^i(c) = 0 \tag{11}$$

It is then straightforward to prove the first welfare theorem for this economy (in fact, for this result we only need that the utility functions are strictly increasing). Hence any competitive equilibrium allocation is the solution to the social planners problem of

$$\max_{\{(c_t^i(s^t))_{i\in I}\}_{t=0,s^t\in S^t}^T} \sum_{i=1}^N \alpha^i u^i(c^i) \tag{12}$$

subject to (5) and (6), for *some* Pareto weights $(\alpha^i)_{i=1}^N$ satisfying $\alpha^i \geq 0$ and $\sum_{i=1}^N \alpha^i = 1$ (see, e.g. MasColell et. al.; this result requires parts of assumption 1). Attaching Lagrange multipliers $\lambda(s^t)$ to the resource constraint and ignoring the non-negativity constraints on consumption we obtain as first order necessary conditions for an optimum

$$\alpha^i \beta^t \pi_t(s^t) U_c^i(c_t^i(s^t), s^t) = \lambda(s^t) \tag{13}$$

for all $i \in I$. Hence for $i, j \in I$

$$\frac{U_c^i(c_t^i(s^t))}{U_c^j(c_t^j(s^t))} = \frac{\alpha^j}{\alpha^i} \tag{14}$$

for all dates t and all states s^t. Hence with a complete set of contingent consumption claims the ratio of marginal utilities of consumption of any two agents is constant across time and states. Also agents, ceteris paribus (i.e. if they had the same utility function and the same preference shock), with higher relative Pareto weights will consume more in every state of the world because the utility function is assumed to be strictly concave.

Assumption 2: *All agents have identical CRRA utility,*

$$U^i(c) = \frac{c^{1-\sigma} - 1}{1-\sigma} \tag{15}$$

with $\sigma \geq 0$ *(for* $\sigma = 1$ *it is understood that utility is logarithmic). Here* σ *is the coefficient of relative risk aversion.*

With this assumption (14) becomes

$$\frac{c_t^i(s^t)}{c_t^j(s^t)} = \left(\frac{\alpha^i}{\alpha^j}\right)^{\frac{1}{\sigma}} \tag{16}$$

i.e. the ratio of consumption between any two agents is constant across time and states. This, in particular, implies that there exist shares $(\theta^i)_{i\in I}$ with $\theta^i \geq 0$ and $\sum_{i\in I}\theta^i = 1$ such that

$$c_t^i(s^t) = \theta^i \sum_{i\in I} y_t^i(s^t) \equiv \theta^i y_t(s^t) = \theta^i c_t(s^t) \tag{17}$$

where $y_t(s^t) = \sum_{i\in I} y_t^i(s^t)$ is the aggregate income in the economy and $c_t(s^t) = \sum_{i\in I} c_t^i(s^t)$ is aggregate consumption. In fact, the shares are given by

$$\theta_i = \frac{1}{\sum_{j=1}^N \left(\frac{\alpha_j}{\alpha_i}\right)^{\frac{1}{\sigma}}} = \frac{\alpha_i^{\frac{1}{\sigma}}}{\sum_{j=1}^N \alpha_j^{\frac{1}{\sigma}}} \geq 0 \tag{18}$$

Note that with logarithmic preferences ($\sigma = 1$) we have that $\theta_i = \alpha_i$, i.e. the share of aggregate consumption an agent i is allocated corresponds to the Pareto weight the planner attaches to this agent.

That is, with separable CRRA utility complete markets imply that individual consumption at each date, in each state of the world is a constant fraction of aggregate income (or consumption). Note that it does *not* imply that individual consumption is constant across time and states of the world, because it still varies with aggregate income (a variation against which no mutual insurance among the agents exists). It also does not imply that consumption among agents is equalized. From (16) we see that the level of consumption of agent i will depend positively on the Pareto weight of that agent.

Let's turn to the characterization of efficient (and hence equilibrium) allocations. The growth rate of consumption between any two dates and states is given from (17) as

$$\log\left(\frac{c_{t+1}^i(s^{t+1})}{c_t^i(s^t)}\right) = \log\left(\frac{c_t(s^{t+1})}{c_t(s^t)}\right) \tag{19}$$

that is, if agents have CRRA utility that is separable in consumption and we have complete markets, then individual consumption growth is perfectly correlated with and predicted by aggregate consumption growth. In particular, individual income growth should not help to predict individual consumption growth once aggregate consumption (income) growth is accounted for. This idea is the basis of all empirical tests of perfect consumption insurance, see e.g. Mace (1991), Cochrane (1991), among others.

To obtain Arrow Debreu prices associated with equilibrium allocations we obtain from the consumer problem of maximizing (4) subject to (9) that

$$\frac{p(s^{t+1})}{p(s^t)} = \beta\frac{\pi_{t+1}(s^{t+1})}{\pi_t(s^t)}\frac{U_c^i(c_{t+1}^i(s^{t+1}))}{U_c^i(c_t^i(s^t))} \tag{20}$$

Under assumption 2 this becomes

$$\frac{p(s^{t+1})}{p(s^t)} = \beta\frac{\pi_{t+1}(s^{t+1})}{\pi_t(s^t)}\left(\frac{c^i_{t+1}(s^{t+1})}{c^i_t(s^t)}\right)^{-\sigma} \quad (21)$$

$$= \beta\frac{\pi_{t+1}(s^{t+1})}{\pi_t(s^t)}\left(\frac{c_{t+1}(s^{t+1})}{c_t(s^t)}\right)^{-\sigma} \quad (22)$$

and hence equilibrium Arrow Debreu prices can be written as functions of aggregate consumption only. We then have the following

Proposition 3 *Suppose allocations* $\{(c^i_t(s^t))_{i\in I}\}^T_{t=0,s^t\in S^t}$ *and prices* $\{p(s^t)\}^T_{t=0,s^t\in S^t}$ *are an Arrow-Debreu equilibrium. Then under assumption 2 the allocation* $\{c_t(s^t)\}^T_{t=0,s^t\in S^t}$ *defined by*

$$c_t(s^t) = \sum_{i\in I} c^i_t(s^t) \quad (23)$$

and prices $\{p(s^t)\}^T_{t=0,s^t\in S^t}$ *is an Arrow Debreu equilibrium for the single agent economy with* $I = 1$ *in which the* ***representative agent*** *has endowment*

$$y_t(s^t) = \sum_{i\in I} y^i_t(s^t) \quad (24)$$

and CRRA preferences.

Obviously the content of this proposition lies in the pricing part. It shows that to derive Arrow-Debreu prices (and hence all other asset prices), with complete markets it is sufficient to study the representative agent economy. This insight is the departure of the consumption-based asset pricing literature as developed in Lucas (1978). The discussion also suggests the steps that are needed to establish a representative agent result.

1. Solve the planning problem with many agents. The first welfare theorem and the Negishi-Mantel algorithm (which you will discusss in detail in the review session) enable us to solve a planning problem that yield allocations that coincide with equilibrium allocation.

2. Use allocations from planner's problem to construct aggregates (such us aggregate consumption or prices). In the simple example above aggregates can be solved easily using feasibility.

3. Show that the same aggregates can be derived solving the equilibrium problem of a fictitious single agent.

Later we will use these three steps explicitly to establish a RA result in a more complex environment. Notice the two key assumptions which are important for the proposition (and for extending the representative agent result in dynamic economies): identical-homothetic preferences and complete markets. In one problem set you will have to study what happens when you relax the identical preferences assumption but keep the complete markets assumption. The key insight is that in complete markets marginal utilities of each and every agent are proportional and thus, for asset pricing purposes, the marginal utility of any agent can be used. Notice though that when preferences are not identical across agents or are not homothetic equalization of marginal utilities no longer implies that consumption of each agent is a fixed share of aggregate endowment (or that the ratio of consumption between agent i and agent j is constant) and thus in order to compute the marginal utility of any agent we need to explicitly compute the consumption of that agent, which will in general depend on the structure of preferences across the economy. So when preferences are not identical across consumers and not homothetic in general the representative consumer result fails and in order to characterize prices or the evolution of aggregate variables one needs to know the entire distribution of resources across the economy.

5 Recommended readings:

- Mas Colell et al., Section 4B
- Ljungqvist and Sargent, Sections 8.1 through 8.11