



Macroeconomic Theory (8107)

Fall 2009, Mini 2

Problem set 3

Due Thursday, Dec 3, in class

Question 1. Optimal consumption with borrowing constraints

Consider an infinitely lived consumer with preferences given by

$$\sum_{t=0}^{\infty} \beta^t \log(c_t)$$

The consumer has 0 initial wealth ($a_0 = 0$) and in each (discrete) time period she receives a constant endowment y . She faces a constant interest rate (at which she can borrow or lend) of r , so her budget constraint can be written as

$$c_t + a_{t+1} = a_t(1 + r) + y$$

also assume that she faces a borrowing constraint of the form

$$a_{t+1} \geq -a$$

Assume the following parameter values: $y = 1, a = 4, r = 0.1$

- Solve and plot the paths for optimal consumption and asset accumulation for the following values of the discount factor β , $[0.5, 0.7, 0.99]$.
- In the last case ($\beta = 0.99$) verify that the transversality condition for optimality (see SLP theorem 4.15) holds at your proposed solution

Question 2. Income fluctuation problem with exponential utility

Consider a consumer with the following utility function

$$u(c) = -\frac{1}{\gamma} E \sum_{t=0}^{\infty} \beta^t u(c_t) e^{-\gamma c}$$

which faces the following income process

$$\begin{aligned} y_t &= \rho_0 + \rho_1 y_{t-1} + \varepsilon_t \\ \varepsilon_t &\rightarrow N(0, \sigma) \\ \rho_0 &> 0, 0 < \rho_1 < 1 \end{aligned}$$

The budget constraint can be written as

$$a_{t+1} = (1+r)a_t + y_t - c_t$$

where $r > 0$ is the interest rate. Assume that in period 0 the consumer starts with assets $a_0 > 0$ and given income $y_0 > 0$.

- Define the value of the income stream of the consumer (in terms of today's wealth a_t)

$$P_t = \frac{1}{1+r} E_t \sum_{j=0}^{\infty} \left(\frac{1}{1+r} \right)^j y_{t+j}$$

solve for P_t as a function of y_t, ρ_0, ρ_1 and r (show your calculations)

- Let

$$\begin{aligned} V(a, y) &= \max_c u(c) + \beta EV(a', y') \\ &\quad s.t. \\ a' &= (1+r)a + y - c \end{aligned}$$

be the value function of the consumer. Show that if the value function has the following form

$$B = \frac{1}{1+r-\rho_1}, D(r) \text{ a fixed function}$$

then the consumption function has the following form

$$c = r(a + By + D(r)) + \frac{1}{\gamma r} \log(1+r)$$

- Show that $D(r)$ has the following form

$$\frac{\rho_0 B}{r} - \frac{1}{\gamma r^2} \left(\log(\beta(1+r)) + \log E(e^{-\gamma r B \varepsilon}) \right) - r \log(1+r)$$

- Now consider the special case in which $\rho_1 = 0$ (i.i.d. shocks). Show that $\Delta c_t = c_{t+1} - c_t$ is equal to

$$\Delta c_t = \frac{r}{1+r} (y_{t+1} - \rho_0) + \frac{1}{\gamma} \left(\log(\beta(1+r)) + \log E(e^{-\gamma r B \varepsilon}) \right)$$

- Show that if $\beta(1+r) = 1$ then individual consumption is a random walk with positive drift (i.e. $c_t = c_{t-1} + \Delta + \Gamma \varepsilon_t$) and solve for Δ .

Question 3. Income and consumption in the PSID

In 2005 the Panel Study of Income Dynamics (PSID) has started collecting comprehensive consumption expenditure data for a panel of US families. Using data from the PSID (<http://psidonline.isr.umich.edu/>) do the following.

- Collect data for family labor income (i.e the sum of labor income of the head, the spouse and of others), family total money income (this include all sources of income) and family expenditures (in this variable you should include food, household furnishings and equipment, clothing and apparel, trips and vacations, recreation, health care, housing and transportation, education and child care) for 2005 and 2007.
- Deflate the three measures using the CPI and divide each family measure by the number of members in the family so to obtain per-capita real measures. Compute growth rates of real per capita income (both measures) and consumption for each family.
- Select only families which report positive measures of all the three quantities in both years and rank families by per-capita real labor income log differences. Divide the ranked families in 10 quantiles (i.e. starting from the ones with lowest growth to the one with the highest growth) and produce the following plot: on the x axis report the average log change of labor income in each decile and on the y axis report the average (in the group) log change in money income and in consumption expenditure.
- Interpret your results using the PIH model studied in class. In particular what does the consumption response tell you about the persistence of the labor income shocks?