



Macroeconomic Theory (8107)

Fall 2009, Mini 2

Problem set 2

Question 1. Distributions under complete and incomplete markets

1. Consider the following version of the economy discussed by Chatterjee. There are two consumers. Consumer 1 starts owning 40% of a representative firm, consumer 2 owns 60% of the firm. Assume that consumers can trade a full set of Arrow securities at time 0. The representative firm solves

$$\begin{aligned} & \max_{\{k_j\}} \sum_{j=0}^{\infty} p_j \left(k_j^\alpha + (1 - \delta) k_j - k_{j+1} \right) \\ & \equiv \max_{\{k_j\}} \sum_{j=0}^{\infty} p_j d_j \end{aligned}$$

where p_j is the price of consumption at date j relative to consumption at date 0. Assume that initial capital is 80% of the steady-state capital. Preferences of both consumer are given by

$$\sum_{t=0}^{\infty} \beta^t \log(c_t - \bar{c})$$

Assume that $\alpha = 0.4, \delta = 0.1, \beta = 0.9, \bar{c} = 0.1$. Solve and plot for the paths, as the economy transits from its initial condition to the steady state, of risk free interest rate, consumption and wealth of both consumers. Also plot the saving rate of both consumers where saving rate is $(s_{it}d_t - c_{it})/s_{it}d_t$

2. Now assume that consumers cannot trade any assets so in each period

$$c_{it} = s_{i0}d_t$$

- (a) Assume that the firm chooses the same capital stock as in part 1. a1) Solve and plot consumption and wealth for both consumers in this case. Would consumer 1 (the poor one) want the firm to choose a faster or slower growth of capital stock? How about consumer a2) Explain and briefly discuss why, in general, when markets are not complete the problem of the firm is not uniquely defined
- (b) Compute the price (in terms of proportional increase in lifetime consumption) that a consumer (before knowing its type) is willing to pay to live in the economy of part 1 rather than in the one of part 2.

Question 2. Optimal consumption with growth

Consider the following deterministic income fluctuation problem

$$\begin{aligned} & \max_{\{c_t, a_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t) \\ & s.t. \\ & c_t + a_{t+1} = (1+r)a_t + y_t \\ & a_{t+1} \geq -\bar{a} \leq 0 \end{aligned}$$

with $u' > 0, u'' < 0, \beta(1+r) = 1, \bar{a} < \infty$ and $\{y_t\}_{t=0}^{\infty} = (1+g)^t, 0 < g < r, a_0 = 0$

- Solve for the the optimal consumption and wealth path as a function of \bar{a} (as warm-up start with the easy case in which $\bar{a} = 0$)
- Now assume that the borrowing constraint has the following form:

$$\frac{a_{t+1}}{y_{t+1}} \geq -\bar{a} < 0$$

Solve for the minimum value of \bar{a} that allows the consumer to maintain a constant consumption for every period t .