University of Minnesota

8107 Macroeconomic Theory, Fall 2009, Mini 2

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Lecture 2. Asset pricing with the representative agent

In this class we'll analyze a commonly used application of application of model economies in which the RA holds, that is asset pricing. The key idea is that aggregate consumption here is a sufficient statistic to price assets. Consider again the basic economy we discussed in the first class i.e. many consumers with identical preferences and endowment of non storable goods. In this economy it is easy to solve for equilibrium allocation using the planning problem. Once the allocation are computed assets can be priced, even though in equilibrium they are not traded (for more details on this see Sargent section 3.5). Here we will just discuss some examples:

Example 1: pricing a one period risk free bond

$$q^1 = \beta E \frac{u'(c')}{u'(c)}$$

Now assume that period utility is given by

$$u(c) = \frac{c^{1-\gamma}}{1-\sigma}$$

then the pricing becomes

$$q^1 = \beta E \left(\frac{c'}{c}\right)^{-\gamma}$$

or writing everything in terms of risk free interest rate R^f

$$R^f = \frac{1}{q^1} = \frac{1}{\beta}\frac{1}{Eg^{-\gamma}} \tag{1}$$

where $g = \frac{c'}{c}$. This pricing equation tells you how interest rate should move with change in expected growth in consumption. As expected growth increases agents would like to borrow to increase their current consumption, but since that is not feasible the real interest rate has to increase to clear credit markets. Notice that in this case σ is the elasticity of interest rate to changes in growth.

Example 2. Pricing a bond that pays a fixed coupon of 1 for 2 periods. here the key thing to notice is that such a bond one period after maturity is just the bond analyzed in the previous case hence the pricing equation can be written as

$$\begin{aligned} q^2 u'(c) &= \beta E u'(c')(1 + q^{1\prime}) \\ q^2 &= \beta E \frac{u'(c')}{u'(c)} + \beta^2 E \frac{u'(c'')}{u'(c)} \end{aligned}$$

Example 3. Pricing an arbitrary stream of future contingent payouts, as for example a defaultable bond, an option or any financial derivative:

$$q_t = \beta E \frac{u'(c_{t+1})}{u'(c_t)} d_{t+1}$$

notic that if you define

$$m_{t+1} = \frac{u'(c_{t+1})}{u'(c_t)}$$

then the condition can be written as

$$\begin{aligned} q_t &= \beta E m_{t+1} d_{t+1} \\ &= \beta E(d_{t+1}) E(m_{t+1}) + \beta cov(m_{t+1}, d_{t+1}) \end{aligned}$$

which reveals the two fundamental components of asset pricing, which are the expected value of the payouts (captured by the first term) and the covariation of the payouts with marginal utility (captured by the second term).Notice for example that if agents are risk neutral then $M_{t+1} = 1$ and only the first component appears and

$$q_t = \beta E(d_{t+1})$$

Example 4. Pricing claims to the stock market. The stock market as a whole can be interpreted as a claim to share of aggregate consumption, i.e. $d_t = c_t$ So the pricing equation in this case is

$$p_t = \beta E\left(u'(c_{t+1})\frac{u'(c_{t+1})}{u'(c_t)}(d_{t+1} + p_{t+1})\right)$$

doing repeated substitutions and taking limits yield

$$p_t = \sum_{j=1}^{\infty} \beta^j d_{t+j} \frac{u'(c_{t+j})}{u'(c_t)} = \sum_{j=1}^{\infty} \beta^j c_{t+j} \frac{u'(c_{t+j})}{u'(c_t)}$$

It is instructive to notice that in the the special case of log utility this reduces to

$$p_t = \frac{1}{u'(c_t)} \sum_{j=1}^{\infty} \beta^j = \frac{\beta c_t}{1-\beta}$$

where the price depends on current consumption but does not depend on future consumption nor on expecred consumption growth. Why?

Notice that in general the pricing depends on 3 elements: preferences, allocations and dividends streams of the assets. In the case of time separable CRRA preference are summarized by 2 parameters, one is the discount factor β the other is the curvature parameter σ (which is at the same time risk aversion and IES)

Example 4. The equity premium puzzle

Rajnish Mehra and Ed Prescott in a paper that appeared in 1985 of the Journal of Monetary Economics pointed to a puzzle i.e. to some pricing implications of the representative agent model that did not square well with observed prices. Assume that a period is a quarter. Let R^s be the annualized return on stocks and R^f be the return on bonds over a quarter. Assume that preferences are time separable CRRA. In this case we can define the so-called stochastic discount factor m_t as

$$m_{t+1} = \frac{u'(c_{t+1})}{u'(c_t)} = \left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}$$

and it is easy to derive the following restriction implied by the asset pricing equations discussed above

$$E_t(m_{t+1}(R^s - R^f)) = 0 \tag{2}$$

now using the definition of covariance we can rewrite (2) as

$$E(R^s - R^f)E(m_{t+1}) = -cov(m_{t+1}(R^s - R^f))$$

or

$$\frac{E(R^s - R^f)}{\sigma_{R^s - R^f}} = -corr(m_{t+1}(R^s - R^f))\frac{\sigma_{m_{t+1}}}{E(m_{t+1})} \tag{3}$$

The left hand side of (2) can be measured directly in the data and it is a common measure of the ratio of returns to risk in asset markets called the Sharpe ratio. In postwar US data, the mean annualized return of stocks over bonds is about 8% with a standard deviation of about 16%, so the Sharpe ratio is about 0.5. Also assume that $\frac{c_{t+1}}{c_t}$ is log-normal i.e. that $\log c_{t+1} - \log c_t$ is normal with mean μ_c and standard deviation σ_c, so that m_t is also log normal with mean $-\gamma\mu_c$ and standard deviation $\gamma\sigma_c$. Notice that the expression $\frac{\sigma_{m_{t+1}}}{E(m_{t+1})}$ is also called the coefficient of variation which in the case of a log normal distribution is equal to $\sqrt{e^{\sigma_c} - 1} \simeq \sigma_c$ so that

$$\frac{E(R^s - R^f)}{\sigma_{R^s - R^f}} = -corr(m_{t+1}(R^s - R^f))\gamma\sigma_c$$

now since $\left|corr(m_{t+1}(R^s - R^f))\right| \leq 1$ we have that

$$\frac{\left|E(R^s - R^f)\right|}{\sigma_{R^s - R^f}} \leq \gamma\sigma_c \tag{4a}$$

Now simply use post war US data to estimate $\sigma_c \simeq 1\%$ which impies that γ must be at least 50 (Notice that value would be even larger if $corr(m_{t+1}(R^s - R^f))$ is less than 1 in absolute value). inequality 4a is a special case of what in finance is usually referred to as Hansen Jagannathan bound. The intuition behind this result is that in order for stock to command a very high premium over bonds it must be that the representative agent really dislike stocks and in this model the only reason why he or she dislikes them is because they pay low returns in state in which its marginal utility is high i.e. $corr(m_{t+1}(R^s - R^f) < 0$ and so they create additional risk and the agent really hates risk (i.e. high risk aversion). One problem with such a high value for γ his is a value that is hard to justify based on studies of individual preferences. But even if you are willing to accept high risk aversion there is an additional problem with picking a very high value for γ, that it also implies very low value for the intertemporal elasticity of substitution. If you go back to equation 1 you see that high value of γ implies extremely high values for R^f because if agents are really unwilling to substitute then it takes an extremely high interest rate to deter them from borrowing in anticipation of future growth. That again can be solved if you are willing to assume a very low value for β, but still a high γ would imply that the interest rate should be extremely sensitive to changes in growth expectations, which we do not really see.