Macroeconomic Theory
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Macroeconomic Theory (8107)

Spring 2008, Mini 1

Problem set 4

Due Thursday, March 11, in class.

Consider a consumer with the following utility function

$$E\sum_{t=0}^{\infty}\beta^t u(c_t)$$
$$u(c) = -\frac{1}{\gamma}e^{-\gamma c}$$

which faces the following income process

$$y_t = \rho_0 + \rho_1 y_{t-1} + \varepsilon_t$$
$$\varepsilon_t \rightarrow N(0,\sigma)$$
$$\rho_0 > 0, 0 < \rho_1 < 1$$

The budget constraint can be written as

$$a_{t+1} = (1+r)a_t + y_t - c_t$$

where $r > 0$ is the interest rate. Assume that in period 0 the consumer starts with assets $a_0 > 0$ and given income $y_0 > 0$.

- Define the value of the income stream of the consumer (in terms of today's wealth a_t)
$$P_t = \frac{1}{1+r}E_t\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j y_{t+j}$$
solve for P_t as a function of y_t, ρ_0, ρ_1 and r (show your calculations)

- Let

$$\begin{aligned} V(a,y) &= \max_c u(c) + \beta E V(a',y') \\ & \quad s.t. \\ a' &= (1+r)a + y - c \end{aligned}$$

be the value function of the consumer. Show that if the value function has the following form

$$\begin{aligned} & \frac{1}{\gamma r} e^{-\gamma r(a+By+D(r))} \\ B &= \frac{1}{1+r-\rho_1}, D(r) \text{ a fixed function} \end{aligned}$$

then the consumption function has the following form

$$c = r(a + By + D(r) + \frac{1}{\gamma r}\log(1+r))$$

- Show that $D(r)$ has the following form

$$\frac{\rho_0 B}{r} - \frac{1}{\gamma r^2}\left(\log(\beta(1+r) + \log E(e^{-\gamma r B\varepsilon})) - r\log(1+r)\right)$$

- Now consider the special case in which $\rho_1 = 0$ (i.i.d. shocks).Show that $\Delta c_t = c_{t+1} - c_t$ is equal to

$$\Delta c_t = \frac{r}{1+r}(y_{t+1} - \rho_0) + \frac{1}{\gamma}\left(\log(\beta(1+r) + \log E(e^{-\gamma r B\varepsilon})\right)$$

- Show that if $\beta(1+r) = 1$ then individual consumption is a random walk with positive drift (i.e. $c_t = c_{t-1} + \Delta + \Gamma\varepsilon_t$) and solve for Δ.