Macroeconomic Theory
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Macroeconomic Theory (8107)

Spring 2008, Mini 1

Problem set 3

Due Thursday, Feb 14, in class. Note we can turn in this problem in group (max 3 persons, no exceptions to this rule)

There are two countries, one good, used for consumption, investment both countries and labor is immobile across countries. A star variable will denote a foreign country variable.

Preferences are given by

$$\sum_{t=0} \sum_{s^t} \beta^t \pi(s^t) u(c(s^t), l(s^t))$$

in the home country and by

$$\sum_{t=0} \sum_{s^t} \beta^t \pi(s^t) u(c^*(s^t), l^*(s^t))$$

in the foreign country, where c and l represent consumption and labor respectively, β is the (common across countries) discount factor and s^t represents the history of events up to time t. A representative firm in each country owns the capital stock and hires labor to operate a technology given by

$$\begin{aligned} y(s^t) &= e^{A(s^t)} F(k(s^{t-1}), l(s^t)) \\ y^*(s^t) &= e^{A^*(s^t)} F(k^*(s^{t-1}), l^*(s^t)) \end{aligned}$$

where $k(s^{t-1})$ is capital stock installed in home country in period t and $A(s^t)$ is total factor productivity. Capital stock evolves according to

$$\begin{aligned} k(s^t) &= (1-\delta)k(s^{t-1}) + x(s^t) - \phi(\frac{x(s^t)}{k(s^{t-1})})k(s^{t-1}) \\ k^*(s^t) &= (1-\delta)k^*(s^{t-1}) + x^*(s^t) - \phi(\frac{x^*(s^t)}{k^*(s^{t-1})})k^*(s^{t-1}) \end{aligned}$$

where x denote investment and ϕ is a convex function capturing the capital stock adjustment costs. Finally the world resource constraint has to hold in every state

$$y(s^t) + y^*(s^t) = x(s^t) + x^*(s^t) + c(s^t) + c^*(s^t)$$

Total factor productivity is an exogenous variables and is governed by autoregressive stochastic processes.

$$\begin{aligned} \begin{pmatrix} A_t \\ A_t^* \end{pmatrix} &= \begin{pmatrix} .98 & 0 \\ 0 & .98 \end{pmatrix} \begin{pmatrix} A_{t-1} \\ A_{t-1}^* \end{pmatrix} + \begin{pmatrix} \varepsilon_t \\ \varepsilon_t^* \end{pmatrix} \\ \begin{pmatrix} \varepsilon_t \\ \varepsilon_t^* \end{pmatrix} &\rightsquigarrow N(0, \Sigma),\ \Sigma = .01 \begin{bmatrix} 1 & .2 \\ .2 & 1 \end{bmatrix} \end{aligned}$$

Assume the following functional forms

$$u(c, l) = \frac{[c^\mu (1-l)^{1-\mu}]^{1-\sigma}}{1-\sigma}$$

$$F(k, l) = k^\alpha l^{1-\alpha}$$

$$\begin{aligned} \phi\left(\frac{x}{k}\right) &= \psi(\frac{x}{k} - \delta)^2 \\ \psi &> 0 \end{aligned}$$

and assume that the parameters are the same in both countries.

1. Take initial conditions $k(s^{-1}), k^*(s^{-1}), A(s^0), A^*(s^0)$ as given and assume that consumers in both countries can trade a full set of AD securities . Define a competitive equilibrium. Write down the first order necessary conditions that characterize equilibrium allocations for this economy. State precisely under what conditions the first order conditions characterizing the competitive equilibrium are equivalent to the first order conditions of a planner which chooses allocation to maximize the equally weighted utility of agents in the two countries.

2. Assume that $A(s^t) = A^*(s^t) = 0$ and $l(s^t) = l^*(s^t) = \bar{l}$ for every s^t.Characterize analytically a symmetric steady state equilibrium i.e. solve for c, k, x, y in both countries as a function of parameters and of $\bar{l}$.

3. Assume that a period in the model is a quarter. Discuss how you would select the values of the parameters β, μ , $\sigma, \alpha, \delta, \psi$. Report you preferred set of parameters.

4. Write down the log-linearized first order conditions which characterize the equilibrium around the steady state, denoting steady state values with bar (i.e. $\bar{c}$), and log deviations from steady state with hats (i.e. $\hat{c}$)

5. Assume the following parameters values
$$\begin{aligned} \beta &= 0.99 \\ \mu &= 0.3 \\ \sigma &= 2 \\ \alpha &= 0.4 \\ \delta &= 0.025 \\ \psi &= 0.8 \end{aligned}$$
solve numerically for the steady state of the model, solve (using a numerical package of your choice) the linear dynamical system which characterize the equilibrium, simulate 20 times the linearized model for 100 periods. Each time HP filter the simulated data. Compute the average cross country correlation of output,investment consumption and employment implied by the model.

6. Now consider a version of the model above in which the only asset which is traded internationally is a non contingent bond. Assume that in the initial period both countries have 0 holding of the bond and argue that the steady state values for c, k, x, y, l in this economy are the same as in the complete markets economy. Write down the log-linearized first order conditions which characterize the equilibrium around the steady state in this economy, solve and simulate the system using the same parameters as in point 5 and recompute the cross country correlations in point 5. Explain why they are different.