University of Minnesota

8107 Macroeconomic Theory, Spring 2008, Mini 2

Fabrizio Perri

Lecture 3. A basic model of risk sharing: the international business cycles model

In this lecture we will consider a basic model of risk sharing, that is the one-good two-countries international real business cycle model, analyzed by Backus, Kehoe and Kydland in their 1992 JPE piece. The model has been widely used to study a variety of issues in international economics, including the transmission of business cycle (i.e. what happens in Europe when the US goes into a recession?) or the current account responses to shocks (i.e. what happen to the US current account when US goes into a recession?). For the purpose of this class we will not focus much on the issues per se (you will see those in the second year) but we will use the model as an example of a risk sharing problem. A risk sharing problem involves 2 or more individuals which face idiosyncratic risk. The normative issue is then how and how much are they able to share the idiosyncratic part of the risk. The positive issue is whether a model in which risk sharing in imperfect can be uses to understand existing features of the world. In order to fully understand bot positive and normative aspects of risk sharing we will consider 4 different market structures, spanning from complete risk sharing to no risk sharing at all: complete markets, stock economy, bond economy and autarky.

1 Preference, technology and resource constraints

There are two countries, one good, used for consumption and investment in both countries and labor is immobile across countries. A star variable will denote a foreign country variable.

Preferences of the representative consumer in each country are given by

$$\sum_{t=0} \sum_{s^t} \beta^t \pi(s^t) u(c(s^t), l(s^t))$$

in the home country and by

$$\sum_{t=0} \sum_{s^t} \beta^t \pi(s^t) u(c^*(s^t), l^*(s^t))$$

in the foreign country, where $\pi(s^t)$ represent the probability of state s^t, β the common discount factor and c and l represent consumption and labor respectively. Notice that here we are already imposing three strong assumptions, one is that preferences are identical across countries, the second is that preferences are time separable, the third is that information sets (as reflected in the $\pi(s^t)$) are also identical. Departing from these assumption has very strong implications for risk sharing.

In each country there is a representative firm which owns the capital stock and hires labor to operate a CRS technology given by

$$\begin{aligned} y(s^t) &= e^{A(s^t)} k(s^{t-1})^\alpha l(s^t)^{1-\alpha} \\ y^*(s^t) &= e^{A^*(s^t)} k^*(s^{t-1})^\alpha l^*(s^t)^{1-\alpha} \end{aligned}$$

where $k(s^{t-1})$ is capital stock installed in home country in period t and $A(s^t)$ is the log of total factor productivity. Capital stock evolves according to

$$\begin{aligned} k(s^t) &= (1-\delta)k(s^{t-1}) + x(s^t) \\ k^*(s^t) &= (1-\delta)k^*(s^{t-1}) + x^*(s^t) \end{aligned}$$

where x denote investment.

We can thus write the home firm objective function as

$$\begin{aligned} &\max_{x(s^t)} \sum_{t=0} \sum_{s^t} Q(s^t) d(s^t) \\ d(s^t) &= y(s^t) - w(s^t) l(s^t) - x(s^t) \end{aligned}$$

where $Q(s^t)$ is the intertemporal price the firm uses to value dividends and $w(s^t)$ is the home wage rate The foreign firm solves analogous problem. Finally the world resource constraint has to hold in every state

$$y(s^t) + y^*(s^t) \geq x(s^t) + x^*(s^t) + c(s^t) + c^*(s^t)$$

2 Complete Markets

In this case we will assume that the domestic households own the domestic firm while foreign households own the foreign firm (complete home bias) but households can internationally trade a full set of one period AD securities. The budget constraint of the representative agent at home is given by

$$d(s^t) + w(s^t) l(s^t) + b(s^t, s^{t-1}) = c(s^t) + \sum_{b(s^{t+1}|s^t)} q(s^{t+1}, s^t) b(s^{t+1}, s^t)$$

where $q(s^{t+1}, s^t)$ and $b(s^{t+1}, s^t)$ are quantities and prices of the AD securities. The foreign agent faces the analogous constraint

$$d^*(s^t) + w^*(s^t) l^*(s^t) + b^*(s^t, s^{t-1}) = c^*(s^t) + \sum_{b(s^{t+1}|s^t)} q(s^{t+1}, s^t) b^*(s^{t+1}, s^t)$$

Market clearing requires

$$b^*(s^{t+1}, s^t) + b(s^{t+1}, s^t) = 0, \text{ for every } s^{t+1}, s^t$$

Summing budget constraints across agents, using the definition of dividends and using the competitive wage it is easy to obtain the world resource constraint. The first order conditions of each agent for the AD securities are

$$\begin{aligned} u_c(s^t) q(s^{t+1}, s^t) &= \beta \pi(s^{t+1}|s^t) u_c(s^{t+1}) \\ u^*_{c^*}(s^t) q(s^{t+1}, s^t) &= \beta \pi(s^{t+1}|s^t) u_{c^*}(s^{t+1}) \end{aligned}$$

where $u_c(s^t) = \frac{\partial u(c(s^t), l(s^t))}{\partial c(s^t)}$, which implies that the ratio of marginal utility of consumption of the two consumers is constant across all dates and all states i.e.

$$\frac{u_c(s^t)}{u^*_{c^*}(s^t)} = \frac{u_c(s^{t+1})}{u^*_{c^*}(s^{t+1})}, \text{ for every } s^t, s^{t+1}, t$$

This condition is usually known as the perfect risk sharing condition, and it is a necessary condition for the allocation to be efficient If this condition is not satified then it is possible to reallocate resources across dates and/or states such that utility of both agents is increased. This condition usually implies pretty strong restriction on observables and it can be tested both in domestic or international setups. For example what does the condition implies if one assumes that $u(c,l) = \frac{c^{1-\sigma}}{1-\sigma} + v(l)$?

In this economy if we normalize the price of consumption in state s^0 to 1 then the value of consumption in state s^t relative to consumption in state 0 is going to be $\beta^t \pi(s^t) \frac{u_c(s^t)}{u_c(s^0)}$ for consumer in country 1. Since we assumed that the domestic firm is owned entirely by domestic consumer it is reasonable to assume that $Q(s^0) = 1$ and $Q(s^t) = \beta^t \pi(s^t) \frac{u_c(s^t)}{u_c(s^0)}$.Taking first order conditions with respect to $k(s^t)$ for the firm under this assumption, we obtain

$$\begin{aligned} \beta^t \pi(s^t) \frac{u_c(s^t)}{u_c(s^0)} &= \sum_{s^{t+1}} \beta^{t+1} \pi(s^{t+1}) \frac{u_c(s^{t+1})}{u_c(s^0)} \left(f_k(s^{t+1}) + 1 - \delta \right) \\ u_c(s^t) &= \beta \sum_{s^{t+1}} \pi(s^{t+1}|s^t) u_c(s^{t+1}) \left(f_k(s^{t+1}) + 1 - \delta \right) \end{aligned}$$

where

$$f_k(s^{t+1}) = \frac{\partial e^{A(s^{t+1})} k(s^t)^\alpha l(s^{t+1})^{1-\alpha}}{\partial k(s^t)}$$

using the perfect risk sharing condition and rearranging we obtain the following two conditions

$$\begin{aligned} \sum_{s^{t+1}} \pi(s^{t+1}|s^t) \left(f_k(s^{t+1}) + 1 - \delta \right) u_c(s^{t+1}) &= \sum_{s^{t+1}} \pi(s^{t+1}|s^t) \left(f_{k^*}(s^{t+1}) + 1 - \delta \right) u_c(s^{t+1}), \ \forall s^t \quad (1) \\ \sum_{s^{t+1}} \pi(s^{t+1}|s^t) \left(f_k(s^{t+1}) + 1 - \delta \right) u^*_{c^*}(s^{t+1}) &= \sum_{s^{t+1}} \pi(s^{t+1}|s^t) \left(f^*_{k^*}(s^{t+1}) + 1 - \delta \right) u^*_{c^*}(s^{t+1}), \ \forall s^t \quad (2) \end{aligned}$$

Conditions (1) and (2) state that the expected marginal value of capital in both countries is equalized for both home and domestic consumers. In other words domestic and foreign consumers fully agree on the investment policies of both firms, thus ownership of the firm is actually irrelevant. In general this condition also tells us that the expected productivity of capital across countries tend to be equalized, so that if, say, country 1, has a positive productivity shock which is expected to last then the expected marginal product of capital in that country will increase and in order to restore equalization capital will flow into that country. This type of economies are easy to solve as the first welfare theorem applies and allocation can be characterized by computing a planning problem. An interesting application is as usual asset pricing. Let $p(s^t)$ be the stock price in time t.In class we will show that $p(s^t) = k(s^t)$. Another set of implications that we can draw from this model and then compare to the data is about the transmission of business cycles (look at the original Backus Kehoe Kydland for this).

We will now turn to economies in which markets are incomplete; nevertheless perfect risk sharing (i.e efficiency) might or might not be achieved. One point you should keep in mind is that complete markets and perfect risk sharing (i.e. efficiency) are not the same thing. The former is a market structure while the latter is a feature of the allocation. The former is a sufficient condition for the latter, but the reverse it is not true.

3 Stock Economy

This is an economy in which consumers can only trade stocks in the domestic and foreign firm. let $\lambda(s^{t-1})$ be the amount of shares of the domestic firm owned by the domestic consumer in state s^t and $\lambda^f(s^{t-1})$ be the share of the foreign firm owned by the domestic consumer in state s^t. The budget constraint of the domestic consumer will be given by

$$\lambda(s^{t-1})\left(d(s^t)+p(s^t)\right)+\lambda^f(s^{t-1})\left(d^*(s^t)+p^*(s^t)\right)+w(s^t)l(s^t)=c(s^t)+\lambda(s^t)p(s^t)+\lambda^f(s^t)p^*(s^t)$$

and the one for the foreign consumer is

$$\lambda^{f*}(s^{t-1})\left(d(s^t)+p(s^t)\right)+\lambda^*(s^{t-1})\left(d^*(s^t)+p^*(s^t)\right)+w^*(s^t)l^*(s^t)=c^*(s^t)+\lambda^{f*}(s^t)p(s^t)+\lambda^*(s^t)p^*(s^t)$$

where λ and λ^f represent holdings of shares of domestic and foreign stock in the home country and λ^* and λ^{f*} are the same variables abroad. Market clearing conditions are given by

$$\begin{aligned}\lambda(s^t)+\lambda^{f*}(s^t) &= 1\\ \lambda^f(s^t)+\lambda^*(s^t) &= 1 \text{ for every } s^t\end{aligned}$$

. This economy seems more realistic than the AD economy as the assets which are actually traded resemble actual assets. An interesting question is though whether and under which conditions in this economy it is possible to replicate the complete markets allocation. Analytically there are not many general results on the topic (one exception is Duffie and Huang, Econometrica, 1985), but there are few interesting examples. Consider a simplified version of the economy we discussed above in which

$$u(c,l)=\log(c)+v(l)$$

then the perfect risk sharing condition implies (assuming a symmetric equilibrium) that

$$\Delta c(s^t)=c(s^t)-c^*(s^t)=0\ \forall s^t$$

now consider the budget constraints of the two consumers together with the market clearing conditions and solve for $\Delta c(s^t)$ to get

$$\begin{aligned}\Delta c(s^t) &= (2\lambda(s^{t-1})-1)\left(d(s^t)+p(s^t)\right)+(1-2\lambda^*(s^{t-1}))\left(d^*(s^t)+p^*(s^t)\right)+(1-2\lambda(s^t))p(s^t)\\ &\quad -(1-2\lambda^*(s^t))p^*(s^t)+w(s^t)l(s^t)-w^*(s^t)l^*(s^t)\\ &= \Lambda(s^{t-1})d(s^t)-\Lambda^*(s^{t-1})d(s^t)-p(s^t)(\Lambda(s^t)-\Lambda(s^{t-1}))\\ &\quad +p^*(s^t)(\Lambda^*(s^t)-\Lambda^*(s^{t-1}))+w(s^t)l(s^t)-w^*(s^t)l^*(s^t)\end{aligned}$$

where we define a renormalized share $\Lambda=1+2\lambda$.

The question we'd like to ask is whether there are sequences for $\Lambda(s^t), \Lambda^*(s^t), p(s^t), p^*(s^t)$ which imply $\Delta c(s^t) = 0$ (i.e. the perfect risk sharing condition) and at the same time constitute an equilibrium for the stock economy. In such sequences are hard to find analytically but there is a special case in which those can be easily found. Remember that $d(s^t) = \alpha y(s^t) - x(s^t)$ and that $w(s^t)l(s^t) = (1-\alpha)y(s^t)$ and focus on an equilibrium in which consider the special case in which $\Lambda(s^t) = \Lambda^*(s^t) = \Lambda$ for every s^t, meaning that in every state each country has the same proportion of domestic and foreign assets and this proportion is constant (obviously this need not to be an equilibrium). But assuming that this indeed is an equilibrium we can write

$$\Delta c(s^t) = \Delta y(s^t)(1 - \alpha + \alpha\Lambda)) - \Lambda \Delta x(s^t)$$

where $\Delta y(s^t) = y(s^t) - y^*(s^t)$ and $\Delta x(s^t) = x(s^t) - x^*(s^t)$. To verify that is indeed an equilibrium with perfect risk sharing we need to ask whether there exists a Λ such that $\Delta c(s^t) = 0$ for every possible realizations of the shocks and the resulting $\Delta y(s^t), \Delta x(s^t)$. This is again hard to prove in general but simplify the economy further and assume that in equilibrium, due to some friction, $x(s^t) = 0$ and $k(s^t) = k^*(s^t) = \bar{k}$ (assume for example that there are infinite adjustment costs and no depreciation). In this case it is easy to show that

$$1 - \lambda = \frac{1}{2\alpha}$$

does the trick, i.e. there is an equilibrium in the stock economy in which each agent holds a constant fraction of foreign assets equal to $\frac{1}{2\alpha}$ and along this equilbrium perfect risk sharing in obtained. Keep in mind that in order to prove that this is an equilibrium one needs to take care of prices, i.e. one has to define prices that support such an equilibrium, verify that those prices exists and verify that, at those prices, agents budget constraints are satisfied. It would be a good exercise for you to do it. The punch line of all this is that perfect risk sharing can, in principle, be obtained even with a more limited set of assets than a full set of Arrow Debreu securities. Notice though that the perfect risk sharing allocation is obtained with a portfolio which is heavily biased toward foreign assets (if one assume $\alpha = 1/3$ then $1 - \lambda = 150\%$!) and that might look a bit strange when compared to actual data on country's holdings of international assets.

4 Bond economy

Let's now consider an even simpler market structure, i.e. the one in which the only assets which is traded internationally is a single non contingent bond. Focusing again for simplicity on the case of no investment the budget constraints of the two agents can be written as

$$\begin{aligned} y(s^t) + b(s^{t-1}) &= q(s^t)b(s^t) + c(s^t) \\ y^*(s^t) + b^*(s^{t-1}) &= q(s^t)b^*(s^t) + c^*(s^t) \end{aligned}$$

where $b(s^t)$ is the quantity of the bond held by each country and $q(s^t)$ is its price. Market clearing conditions are

$$b(s^t) + b^*(s^t) = 0 \text{ for every } s^t$$

We will now show that in this simple market structure departures from perfect risk sharing can be, comparing them with those observed in the previous economy, substantial. In order to show this let's consider the log-linearized first order conditions for the bond in both countries (again focus on log separable utility)

$$\begin{aligned}\hat{q}(s^t) &= E(\hat{c}(s^t) - \hat{c}(s^{t+1}) \\ \hat{q}(s^t) &= E(\hat{c}^*(s^t) - \hat{c}^*(s^{t+1})\end{aligned}$$

where $\hat{c}$ denotes a percentage deviation from steady state. Combining them one gets that

$$E(c(s^{t+1}) - c^*(s^{t+1})) = \hat{c}(s^t) - \hat{c}^*(s^t) \tag{3}$$

Note that the term $\hat{c}(s^t) - \hat{c}^*(s^t)$ represent the difference in consumption between the two countries expressed as percentage of the steady state (where a negative number means country 2 has a greater consumption). Equation .(3) tells us that this object is a random walk so it is going to be arbitrarily larger (in absolute value) meaning that the differences in consumption (and marginal utility) between the two countries are going to get arbitrarily larger with positive probability. The remarkable fact about this is that this is going to happen regardless of the process for $y(s^t)$. Obviously the variance of the innovation of the random walk is going to depend on the persistence properties of the shock, but the fact that the difference between the two countries is getting arbitrarily larger is independent from the persistence properties of the shock. This property tells us that a simple bond is not a very good instrument to share risk in the long run: this is not surprising as a non contingent bond, by its nature, is a good instrument to smooth consumption over time but not across states of the world. Another important difference between bond economies and economies with complete markets is the presence of wealth effects. In complete markets economies, because of the complete risk sharing property, there are no wealth effects in response to shock, i.e. the releative wealth of the two residents is fixed at time 0. In incomeplete markets shocks alter the relative wealth of the residents of the two countries. If for example country 1 is hit by a favorable shock its relatiove wealth increase and this will affect, for example, its decision to supply labor. Consider for example the case in which preferences are given by

$$u(c, l) = \log(c) + \log(1 - l)$$

and production is given by the simple

$$y = Ak^{\alpha}l^{1-\alpha}$$

Labor market equilibrium conditions (which hold regardless of market structure) yield

$$\frac{1}{c}(1-\alpha)Ak^{\alpha}l^{-\alpha} = \frac{1}{1-l}$$

which linearized (assuming that $\hat{k} = 0$) yields

$$\hat{l} = (\frac{1}{\alpha + \frac{\bar{l}}{1-\bar{l}}}) \left(\hat{A} - \hat{c}\right)$$

which shows that the labor supply response to a given productivity shock is dampened by the consumption response to the same shock. In other words when a country is hit by a positive productivity shock the substitution effect leads to residents to supply more labor but the wealth effect leads residents to consume more leisure and thus to lower labor supply. Now with complete markets relative wealth effects are 0 hence the differential respnse of labor supply to differential productivity shocks is maximized, i.e. countries with high productivity tend to work more. With incomplete markets countries which receive a positive productivity shocks also receive a positive relative wealth effect, thus increase labor supply less and thus relative labor supply will respond less to differential productivity. This in general will lead to more correlated labor supplies.