International Recessions

By Fabrizio Perri and Vincenzo Quadrini*

September 2017

Macro developments leading up to the 2008 crisis displayed an unprecedented degree of international synchronization. Before the crisis all G7 countries experienced credit growth, and around the time of the Lehman bankruptcy they all faced sharp and large contractions in both real and financial activity. Using a two-country model with financial frictions we show that a global liquidity shortage induced by pessimistic self-fulfilling expectations can quantitatively generate patterns like those observed in the data. The model also suggests that with more international financial integration crises are less frequent but, when they hit, they are larger and more synchronized across countries.

Keywords: Credit shocks, global liquidity, international co-movement

JEL classification: F41, F44, G01

* Perri: Research Division, Federal Reserve Bank of Minneapolis, 90 Hennepin Avenue, Minneapolis, MN 55480-0291 (email: fperri@umn.edu); Quadrini: Department of Finance and Business Economics, Marshall School of Business, University of Southern California, 701 Exposition Boulevard, HOH 715, Los Angeles, CA 90089 (email: quadrini@usc.edu). We thank Mark Aguiar and three anonymous referees for excellent comments, Philippe Bacchetta, Ariel Burstein, Fabio Ghironi, Jean Imbs, Anastasios Karantounias, Thomas Laubach, Enrique Martínez-García, Dominik Menno, Paolo Pesenti, Xavier Ragot, Etsuro Shioji, and Raf Wouters for thoughtful discussions, and seminar participants at several institutions and conferences for very useful comments and suggestions. Perri acknowledges financial support from the European Research Council under Grant 313671. Quadrini acknowledges financial support from NSF Grant 1460013. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System. The authors declare that they have no relevant or material financial interests that relate to the research described in this paper.

One of the most striking features of the 2008 crisis is that in the midst of it—during the quarter following the Lehman bankruptcy—all major industrialized countries experienced extraordinarily large and synchronized contractions in both *real* and *financial* aggregates. Motivated by this evidence, we develop a simple theory of financial crises in open economies, aiming to make two contributions. The first is to argue that the 2008 crisis could have been the result of a global liquidity shortage induced by pessimistic self-fulfilling expectations. We do so by showing that crisis patterns predicted by our theory are quantitatively consistent with many features of the macro-economy observed before and during the 2008 crisis in the U.S. and other G7 countries. The second contribution is to show how international financial integration affects the probability and the size of crises. In particular with more international financial integration crises are less frequent but, when they hit, they are larger and more synchronized across countries. This finding can have important normative implications, in light of the recent policy debate on the desirability of capital markets integration.

Our analysis is based on a two-country model where firms in both countries use credit to finance hiring and investment, and where the availability of credit depends on the value of collateral, that is, the resale price of assets. The value of collateral is endogenous in the model and depends on the market liquidity (access to credit) which in turn depends on the value of collateral. This interdependence between the value of collateral and liquidity creates the conditions for which the tightness of credit constraints can emerge endogenously as multiple self-fulfilling equilibria.

In 'good' equilibria, the market expects high resale prices for the assets of defaulting firms, which allows for looser borrowing constraints. As a result of the high borrowing capacity, firms are not liquidity constrained and ex post there are firms with the required liquidity to purchase the assets of defaulting firms. This keeps the resale price high and rationalizes, ex post, the ex ante expectation of high collateral values. The higher availability of credit in good equilibria also means that firms borrow more. As credit expands, however, a 'bad' equilibrium could emerge if market expectations about the resale price of assets change and turn pessimistic. Expectations of a low resale value implies that firms face tighter borrowing limits and are liquidity constrained. Because firms are liquidity constrained, there are no firms capable of purchasing the assets of defaulting firms and, as a result, the resale price is low. This rationalizes the expectation of

low prices, leading to 'bad' equilibria characterized by globally reduced credit, deleveraging, and sharply depressed real activity. Financial integration implies that the prices of collateral are equalized across countries, and hence credit conditions are also equalized. It is through this mechanism that the crisis becomes global and displays a high degree of real and financial synchronization.

The theory of endogenous financial booms and busts is important in two respects. First, with endogenous credit shocks the model generates cross-country co-movement not only in real variables but also in financial aggregates. To show this, we first study a version of the model in which country-specific credit conditions change exogenously. If financial markets are integrated, an exogenous tightening of credit in one country depresses employment and output in both countries. However, while the country hit by the shock experiences a credit crunch, the other country experiences a credit boom. Therefore, unless exogenous credit shocks are correlated across countries, the model would not generate financial synchronization. We then show that by making credit conditions endogenous, the model generates synchronized movements in both real and financial variables. This result supports the view that a self-fulfilling, global liquidity shortage, rather than isolated country-specific shocks, is important for understanding the 2008 crisis.

Second, the endogeneity of credit booms and busts allows us to assess how the probability and depth of crises change when financial markets get more integrated. Since a self-fulfilling crisis requires a high degree of coordination in expectations, the likelihood of coordination decreases when markets are integrated: an integrated market is a larger market that requires the coordination of more agents. But as the probability of a crisis decreases, the incentive to leverage increases. Thus with integrated financial markets crises are less frequent, but their macro consequences are bigger.

In the final part of the paper we evaluate the quantitative importance of liquidity induced crises by calibrating the model to the United States and other G7 countries. The simulation over the period 1995-2012 shows that the model captures several features of real and financial data not only during the crisis but also in the period that preceded the crisis. The setup also helps us understand a number of features that are hallmarks of financial crises in general. In particular, the model generates (i) asymmetric dynamics of real variables in credit booms (slow

growth) and credit crashes (sharp contraction), (ii) countercyclical labor productivity, and (iii) crises that are more severe when they arise after a long period of credit expansion. However, the model does not capture the sluggish recovery after the crisis. This suggests that a liquidity shortage can be responsible for the initial collapse in economic activity typical of a financial crisis, but additional mechanisms are needed to understand the sluggish recovery that typically follows the crisis.

One important observation concerning the international dimension of the recent crisis is that employment was hit particularly hard in the United States but, at least initially, not in the other G7 countries. Also, labor productivity did not change significantly in the United States but declined in the other G7 countries. A related observation is that the 'labor wedge' increased significantly in the United States but did not change substantially in other G7 countries (see, for example, Ohanian 2010). Our baseline model with symmetric countries does not capture these cross-country differences. However, in the extension with cross-country heterogeneity in labor rigidities (more flexibility in the United States and less flexibility in other G7 countries), the model can also generate the heterogeneous responses of employment, productivity, and labor wedge.

The paper is related to the large literature on international co-movement. The literature broadly focuses on two channels. The first is based on the existence of global or common shocks, that is, exogenous disturbances that are correlated across countries. The second explanation is based on the international transmission of country-specific shocks (for example through investment). In this paper, we show that credit shocks generate co-movement for both reasons: *exogenous* credit shocks spill over from one country to the other, and *endogenous* credit shocks will appear to the econometrician like a common shock or a global factor. Recent contributions that analyze the role of financial markets for the international co-movement observed during the 2007-2009 crisis include Dedola and Lombardo (2010), Devereux and Yetman (2010), Devereux and Sutherland (2011), Kollmann, Enders, and Müller (2011), and Kollmann (2013).

The role of credit shocks for macroeconomic fluctuations has been recently investigated primarily in closed economy models.[1] In this paper, instead, we study the international implications

[1]Examples are Guerrieri and Lorenzoni (2011), Gertler and Karadi (2011), Jermann and Quadrini (2012), Goldberg (2013), Khan and Thomas (2013), Liu, Wang, and Zha (2013), Bacchetta, Benhima, and Poilly (2014), and Christiano, Motto, and Rostagno (2014). There is also a long list of papers where the financial sector plays

of these shocks and provide a micro foundation based on self-fulfilling expectations. Our theory is in line with the idea of liquidity crises resulting from multiple equilibria outcomes as discussed in Lucas and Stokey (2011) and it shares some similarities with models of bubbles as in Kocherlakota (2009), Martin and Ventura (2012), and Miao and Wang (2017).

The idea that multiple equilibria can emerge in models in which the availability of credit depends on the value of collateral assets has been first proposed by Shleifer and Vishny (1992) and, more recently, by Benmelech and Bergman (2012) and Liu and Wang (2014). These studies, however, consider only closed economy models. Our paper shows that multiple equilibria are also important for capturing the international synchronization of recessions and their severity. In this respect, it relates to the literature studying the sources of macroeconomic co-movement and international transmission of shocks, starting with Backus, Kehoe, and Kydland (1992). A recent study by Bacchetta and Van Wincoop (2016) also proposes a model with multiple equilibria that generates international co-movement. The mechanism developed in their model is based on self-fulfilling expectations about aggregate demand.

A central feature of our model is that financial constraints are 'occasionally binding'. Mendoza (2010), Bianchi (2011), and Bianchi and Mendoza (2013) also study economies with occasionally binding constraints but do not investigate the issue of international co-movement. Occasionally binding constraints are also central to Brunnermeier and Sannikov (2014) and Arellano, Bai, and Kehoe (2012) but their analysis is limited to productivity shocks (level and volatility) and to closed economies. Occasionally binding constraints are central to our setup not only because they generate highly nonlinear dynamics but, more importantly, because they are essential to generating multiple equilibria.

The paper is organized as follows. Section I documents some stylized facts about the crisis. We then describe the theoretical framework starting in Section II with a simpler version of the model without capital accumulation and *exogenous* credit shocks. After showing that exogenous credit shocks do not generate co-movement in the flows of credit, we extend the model in Section III to allow for multiple equilibria and *endogenous* credit shocks. In this section we also show how financial integration affects the likelihood and depth of financial crises. Section IV adds

a role in the propagation of other nonfinancial shocks. Especially interesting are theories based on time-varying uncertainty as in Arellano, Bai, and Kehoe (2012) and on interbank crises as in Boissay, Collard, and Smets (2013).

capital accumulation and conducts the quantitative analysis. Section V concludes.

I Stylized Facts

We now present some facts about international co-movement during the 2007-2009 crisis. Figure 1 plots the GDP dynamics for the G7 countries during the six most recent US recessions. In each panel we plot, for each country, the percentage deviations from the level of GDP in the quarter preceding the start of the US recession. Comparing the bottom right panel of the figure with the other panels shows that the 2007-2009 recession and, in particular, the period following the Lehman crisis, stands out in terms of both depth and macroeconomic synchronization. In none of the previous recessions did GDP fall so much and in all countries.

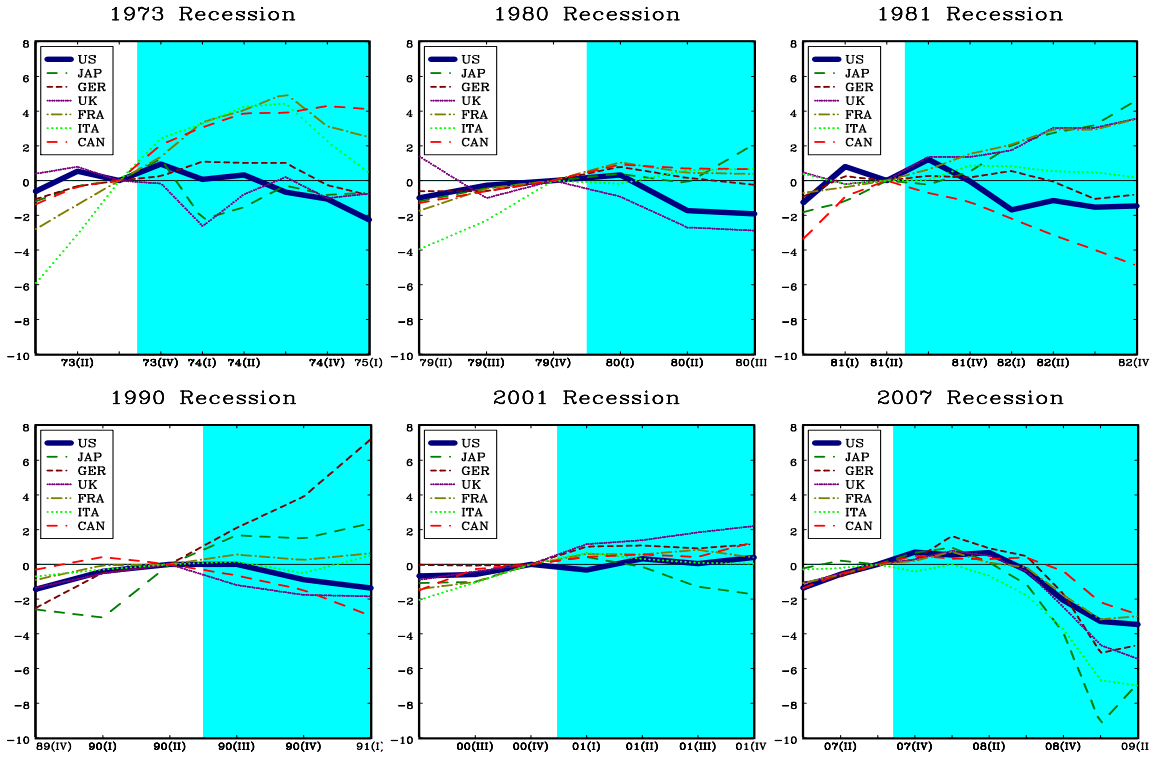

Figure 1: Dynamics of GDP in the G7 countries during the six most recent US recessions

Note: All series normalized to 1 in the quarter preceding the start of the US recession (NBER recession dates).

Another way to illustrate the increased international co-movement associated with the recent

crisis is provided in Figure 2. This figure plots the average bilateral correlations of 10-year rolling windows of quarterly GDP growth between all G7 countries. Two standard deviation confidence bands are also plotted. During the last two quarters of 2008 the average correlation jumped from 0.3 to 0.7 and the sample standard deviation fell significantly. This confirms that the 2007-2009 period stands out in the postwar era as a time of exceptional high co-movement for all developed countries, a point also emphasized by Imbs (2010), among others.

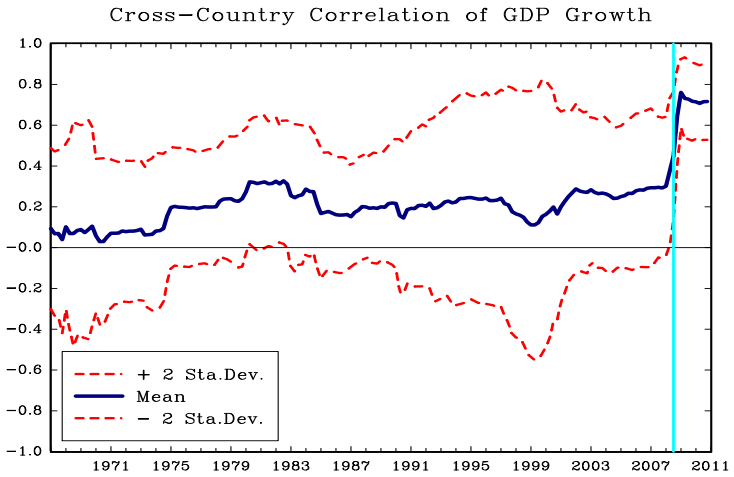

Figure 2: Bilateral rolling correlations of GDP growth for G7 countries

Note: Each correlation is computed over a 10-year window of quarterly GDP growth. The x-axis is the most recent date in the window. The vertical line denotes the third quarter of 2008 (Lehman's bankruptcy).

The high degree of international co-movement between the United States and other major industrialized countries is also observed in other real and financial variables. Figure 3 plots GDP, consumption, investment, and employment in the period 2005-2010 for the United States and an aggregate of the other countries in the G7 group (G6 from now on). The figure highlights that, after the Lehman crisis, GDP, consumption, and investment were all hit hard in both the United States and the G6. Employment also declined in the US and abroad, even though the US decline was much larger than the decline in the G6. We will discuss later the different response of employment observed in the US and other G7 countries.

Figure 4 plots the dynamics of some financial variables. The first panel shows the growth rate of real debt in the private sector for the US and the G6 aggregate. Data for this variable is

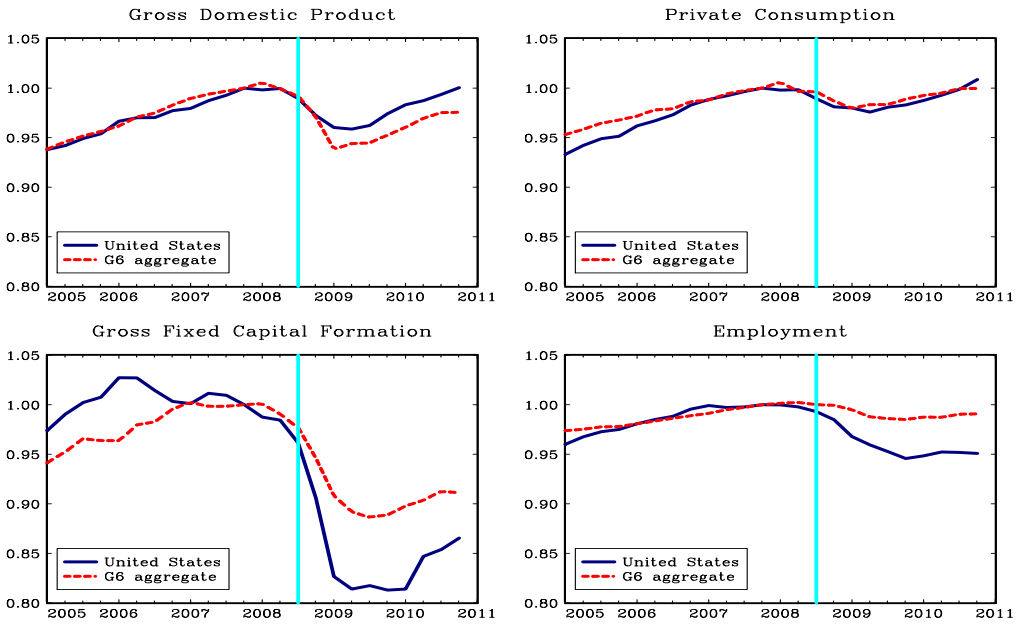

Figure 3: GDP, consumption, investment, and employment: the US and the G6, 2005-2010

Note: Data for GDP, consumption, and investment are from OECD Quarterly National Accounts in PPP constant dollars. Data for employment are from OECD Main Economic Indicators. All series are normalized to 1 in the first quarter of 2007. The vertical line denotes the third quarter of 2008 (Lehman's bankruptcy).

available only annually. The panel shows that the growth in private debt declined significantly going into the crisis in both the US and other G7 countries. The second panel shows a similar pattern for net real debt in the nonfinancial business sector. Net business debt is defined as gross debt minus a measure of liquid assets held by the sector. This series is available quarterly but not for the whole private sector, which explains why we report it separately from the private debt series shown in the first panel.[2]

Indicators of credit market conditions based on credit volumes have been criticized because they do not take into account that a credit crunch might induce firms to draw on existing credit

[2]Private debt used in the first panel is from the OECD statistics database. Net business debt used in the second panel comes from different sources. US net debt is for the nonfinancial business sector from the Flow of Funds Accounts. The series for the G6 is the sum of net debt (in constant PPP dollars) for the corporate nonfinancial sector in the euro area, Japan, and Canada. Thus, the series does not correspond exactly to the series for the G6 aggregate because data for the United Kingdom are not available and it includes Euro countries that are not in the G7 group. Net debt is defined as credit market instruments (gross) minus liquid assets (foreign deposits, checkable deposits and currency, savings deposits, money market funds, securities RPs, commercial paper, treasury securities, agency and GSE-backed securities, municipal securities, and mutual fund shares).

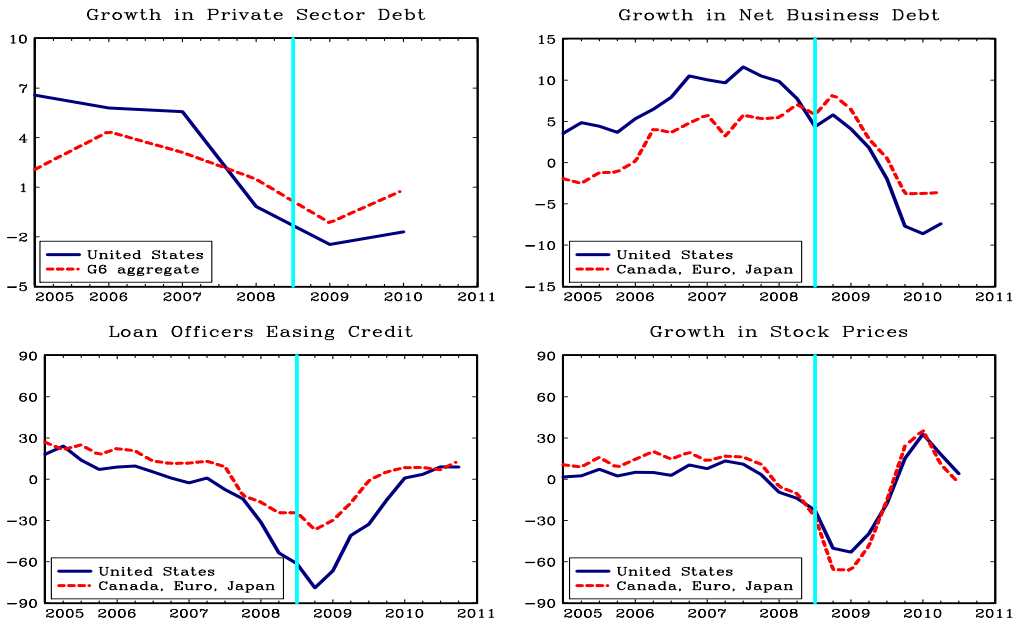

Figure 4: Credit conditions and stock market: the US and the G6, 2005-2010

Note: The vertical line denotes the third quarter of 2008 (Lehman's bankruptcy)

lines, so the distress does not immediately show up in quantities (see, for example, Gao and Yun 2009). For this reason, the bottom left panel of Figure 4 reports a different indicator of credit market conditions. The indicator is not based on volumes of credit but on opinion surveys of senior loan officers of banks. The plotted index is the percentage of banks that relaxed the standards to approve commercial and industrial loans minus the percentage of banks that tightened the standards. Thus, a negative number represents a tightening of credit.[3] As can be seen from the figure, the index shows a credit tightening that starts before the decline in credit growth. Finally, the bottom right panel of Figure 4 plots the growth rate of stock prices in the United States and in the G6 aggregate. The panel documents the massive and synchronous decline in stock prices that took place during the crisis.[4]

[3]The US series is from the Federal Reserve Board (Senior Loan Officers Opinions Survey). The G6 series is based on similar surveys released by the European Central Bank (ECB Bank Lending Survey), Bank of Japan (Senior Loan Officer Opinion Survey), and Bank of Canada (Senior Loan Officers Opinions Survey). It is computed as the weighted (by overall debt) average of the indices for the euro area, Japan, and Canada. The indices are typically reported with the inverted sign (representing the percentage of officers tightening credit standards).

[4]Stock prices for the United States are the MSCI BARRA US stock market index, and stock prices for the G6 countries are computed using the MSCI BARRA EAFE+Canada index which is an average of stock prices in advanced economies except the US economy.

The key lesson we learn from Figure 4 is that, right around 2008, credit conditions moved from strongly loose to strongly tight both in the United States and in the G6 countries.

A final observation relates to the asymmetry between real and financial variables in the expansion phase before the crisis and the collapse during the crisis. The top left panel of Figure 4 shows that, in the years preceding the crisis, debt experienced significant growth. Figure 3, instead, shows that the growth in real variables has been moderate. During the crisis period, however, all variables, both real and financial, contracted sharply. This feature is not unique to the 2007-2009 financial crisis. Several authors have observed that many historical episodes of credit booms are not associated with much faster growth in real economic activity. However, when a credit boom experiences a sudden stop, the reversal is often characterized by sharp macroeconomic contractions. See, for example, Reinhart and Rogoff (2009), Claessens, Kose, and Terrones (2011), and Schularick and Taylor (2012).

The facts presented in this section—high international co-movement in real and financial variables during the crisis, large employment (for the United States), and asymmetry between the precrisis phase and the crisis phase—cannot be easily explained with a standard workhorse international business cycle model. In the next sections we propose a theoretical framework with endogenous credit shocks that helps us understand these facts.

II Model with Exogenous Credit Shocks

We start with a simple model without capital accumulation and with *exogenous* credit shocks. The model provides intuition for the key financial mechanism through which changes in the availability of credit affect employment and the real sector of the economy. However, while the model generates cross-country co-movements in real variables in response to credit shocks, it does not generate co-movement in financial aggregates. We will then extend the setup with *endogenous* credit shocks which allow the model to generate co-movement also in financial variables.

There are two types of atomistic agents: a mass 1 of workers and a mass ω of investors. The relative sizes of workers and investors are irrelevant for the equilibrium properties of the model but will affect the computation of the welfare consequences of financial integration which we will

report later in the paper. Only investors have access to the ownership of firms whereas workers can only save in the form of bonds (market segmentation). Investors and workers have different discount factors: β for investors and $\delta > \beta$ for workers. As we will see, the different discounting implies that in equilibrium firms borrow from workers.[5] To facilitate the presentation we describe first the closed-economy version of the model.

A Investors and Firms

Investors have lifetime utility $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$. They are the owners of firms and can trade shares with other investors. We assume that the mass of firms is fixed and equal to 1. Denoting by n_t the shares of firms held by an individual investor, D_t the aggregate dividends paid by all firms, and P_t the ex-dividend price of shares, the problem solved by an investor can be written recursively as

$$\Omega(\mathbf{s}_t, n_t) = \max_{c_t, n_{t+1}} \left\{ u(c_t) + \beta \mathbb{E}_t \Omega(\mathbf{s}_{t+1}, n_{t+1}) \right\} \tag{1}$$

subject to:

$$n_t(D_t + P_t) = c_t + n_{t+1} P_t,$$

where $\Omega(\mathbf{s}_t, n_t)$ is the value function for the investor which depends on the aggregate states $\mathbf{s}_t$ (defined later) and the shares of firms n_t (individual state). We are assuming that investors do not borrow or save in the form of bonds.

The first order conditions for the investor's problem return the typical Euler equation $u_c(c_t)P_t = \beta \mathbb{E}_t u_c(c_{t+1})(D_t + P_t)$, where the subscript in the utility function denotes derivatives. Investors are homogeneous and they earn only dividend incomes. Therefore, in equilibrium we have $n_t = n_{t+1} = 1/\omega$ and $c_t = D_t/\omega$, where ω has been defined earlier as the fixed mass of investors (while the mass of firms is 1). We can then express the equilibrium price of a share as $P_t = \beta \mathbb{E}_t[u_c(D_{t+1}/\omega)/u_c(D_t/\omega)](D_{t+1} + P_{t+1})$. This shows that investors discount future divi-

[5] Several mechanisms have been proposed in the literature to generate a borrowing incentive for firms: tax deductability of interests, uninsurable idiosyncratic risks for lenders, bargaining of wages, and so on. Since the specific mechanism that leads to the pecking order of debt is not important for our results, we simply assume different discounting as in Kiyotaki and Moore (1997).

dends by $m_{t+1} = \beta u_c(D_{t+1}/\omega)/u_c(D_t/\omega)$. Since firms operate on behalf of investors, this will also be the discount factor used by firms. In what follows we assume that the utility of investors takes the CES form so that ω cancels out. The discount factor is then $m_{t+1} = \beta u_c(D_{t+1})/u_c(D_t)$.

Firms operate the production function $F(h_t) = \bar{k}h_t^{\nu}$, where $\bar{k}$ is the 'fixed' input of capital, h_t is the variable input of labor, and $\nu < 1$ implying decreasing returns in the variable input.

Firms start the period with *intertemporal* debt b_t. Before producing, they choose labor input h_t, dividends d_t, and next period debt b_{t+1}. We use the small letter d_t to denote the dividends paid by an 'individual' firm while in the investor's problem we used the capital letter D_t to denote the 'aggregate' dividends paid by all firms. Whenever necessary, we will use this notation throughout the paper (small letters for individual variables and capital letters for aggregate variables). Denoting by R_t is the gross interest rate, the budget constraint is

$$b_t + w_t h_t + d_t = F(h_t) + \frac{b_{t+1}}{R_t}. \tag{2}$$

The payments of wages, $w_t h_t$, dividends, d_t, and current debt net of the new issue, $b_t - b_{t+1}/R_t$, are made before the realization of revenues. Thus, the firm faces a cash flow mismatch. To cover the cash mismatch, the firm contracts the intraperiod loan $x_t = w_t h_t + d_t + b_t - b_{t+1}/R_t$, which is repaid at the end of the period after the realization of revenues.[6] Using the budget constraint (2), we can see that the intraperiod loan is equal to the revenue, that is, $x_t = F(h_t)$.

Debt contracts are not perfectly enforceable because firms can default. Default takes place at the end of the period before repaying the intraperiod loan. At this stage, a firm holds the revenue $F(h_t)$ which can be diverted. If the firm defaults, the lender has the right to liquidate its assets. However, after the diversion of $F(h_t)$, the only remaining asset is the physical capital $\bar{k}$. Suppose that the liquidation value of capital is $\xi_t \bar{k}$, where ξ_t is stochastic. Then, to ensure that the firm does not default, the lender imposes the enforcement constraint

$$x_t + \frac{b_{t+1}}{R_t} \leq \xi_t \bar{k}. \tag{3}$$

The left-hand side terms are the total liabilities of the firm at the end of the period (intraperiod and intertemporal). The right-hand side term is the liquidation value of firm's capital. The

[6] As an alternative we could assume that firms cannot borrow with intraperiod loans but they can carry cash from the previous period. In this case, firms would carry cash since this is the only way to make the payments before the realization of revenues. The explicit consideration of cash would not change the key properties of the model but would complicate the numerical solution because it adds another state variable.

constraint is derived under the assumption that the firm has the whole bargaining power in the renegotiation of the debt as in Hart and Moore (1994) and can distribute the diverted liquidity as dividends to shareholders. The formal derivation is provided in Appendix A.

To illustrate the role played by fluctuations in ξ_t, consider a preshock equilibrium in which the enforcement constraint is binding. Starting from this equilibrium, suppose that ξ_t decreases. This forces the firm to reduce either the dividends, the input of labor, or both.

To see why, let's start with the assumption that the firm does not change the input of labor h_t. This implies that the intraperiod loan also does not change because $x_t = w_t h_t + d_t + b_t - b_{t+1}/R_t = F(h_t)$. Consequently, the only way to satisfy the enforcement constraint (3) is by reducing the intertemporal debt b_{t+1}. We can then see from the budget constraint (2) that the reduction in b_{t+1} requires a reduction in dividends. Thus, the firm is forced to substitute debt with equity. Alternatively, the firm could keep the dividends unchanged and reduce the intraperiod loan $x_t = F(h_t)$. This would also ensure that the enforcement constraint is satisfied but it requires the reduction in the input of labor. Therefore, after a reduction in ξ_t, the firm faces a trade-off: paying lower dividends or cutting employment. The optimal choice depends on the relative cost of changing these two margins, which, as we will see, depends on the stochastic discount factor $m_{t+1} = \beta u_c(d_{t+1})/u_c(d_t)$.[7]

Firm's Problem: The optimization problem of the firm can be written recursively as

$$V(\mathbf{s}_t; b_t) = \max_{d_t, h_t, b_{t+1}} \left\{ d_t + \mathbb{E}_t m_{t+1} V(\mathbf{s}_{t+1}; b_{t+1}) \right\} \tag{4}$$

subject to:

$$b_t + d_t = F(h_t) - w_t h_t + \frac{b_{t+1}}{R_t} \tag{5}$$

$$F(h_t) + \frac{b_{t+1}}{R_t} \leq \xi_t \bar{k}, \tag{6}$$

where $\mathbf{s}_t$ are the aggregate states (as specified below).

[7]Movements in ξ_t are consistent with Eisfeldt and Rampini (2006) who suggest that the liquidity of capital must be procyclical in order to match the observed reallocation.

The enforcement constraint takes into account that the intraperiod loan is equal to the firm's output, that is, $x_t = w_t h_t + d_t + b_t - b_{t+1}/R_t = F(h_t)$. The firm discounts future payments by m_{t+1}, which is the discount factor we derived earlier from the investor's problem (1). This factor is taken as given by an individual firm because firms are atomistic and investors hold a diversified portfolio of shares. The assumption that firms are atomistic also implies that they take as given all prices when solving the individual problem. The first order conditions, derived in Appendix B, take the form

$$R_t \mathbb{E}_t m_{t+1} = 1 - \mu_t, \tag{7}$$

$$F_h(h_t) = \frac{w_t}{1 - \mu_t}, \tag{8}$$

where μ_t is the Lagrange multiplier associated with the enforcement constraint.

Equations (7) and (8) are key to understanding how financial shocks affect economic activity and in particular employment. Firms borrow resources to finance dividend payments and labor. When a financial shock hits, that is, ξ falls, the shadow value of these resources (μ) increases. Equation (7) then shows that this leads to a reduction in dividend payments with the consequent decline in the stochastic discount factors of investors. This, in turn, generates a decline in the stock market. Equation (8) shows that an increase in the shadow value of resources causes a decline in the demand for labor which, in general, causes a fall in employment.[8]

B Closing the Model and General Equilibrium

The representative worker maximizes the lifetime utility $\mathbb{E}_0 \sum_{t=0}^{\infty} \delta^t U(c_t, h_t)$, where c_t is consumption, h_t is labor, and δ is the intertemporal discount factor. It will be convenient to assume that the period utility takes the form $U(c_t, h_t) = \ln(c_t) - \alpha h_t^{1+1/\eta}/(1 + 1/\eta)$.

Workers supply labor at the competitive wage w_t and can save by holding bonds issued by firms. They can also trade state-contingent claims with other workers. However, they cannot trade contingent claims with firms. This assumption is essential to maintain the segmentation

[8]Recent empirical works by Bentolila et al. (2013), Chodorow-Reich (2014), Greenstone, Mas, and Nguyen (2014), among others, find evidence, both in the US and in Europe, that firms with shortage of credit do cut employment, supporting the mechanism highlighted here.

of financial markets and the relevance of financial frictions.[9]

Denote by $a_{t+1}(\mathbf{s}_{t+1})$ the units of consumption goods received at time $t+1$ by domestic workers if the aggregate states are $\mathbf{s}_{t+1}$. The budget constraint is

$$w_t h_t + b_t + a_t = c_t + \frac{b_{t+1}}{R_t} + \int_{\mathbf{s}_{t+1}} a_{t+1}(\mathbf{s}_{t+1}) q(\mathbf{s}_t; \mathbf{s}_{t+1})/R_t,$$

where $q_t(\mathbf{s}_{t+1})/R_t$ is the unit price for the contingent claims.

Given the specified utility, the first order conditions for labor, h_t, next period bonds, b_{t+1}, and foreign claims, $a_{t+1}(\mathbf{s}_{t+1})$, are

$$\alpha h_t^{\frac{1}{\eta}} c_t = w_t, \tag{9}$$

$$\delta R_t \mathbb{E}_t \left(\frac{c_t}{c_{t+1}} \right) = 1, \tag{10}$$

$$\delta R_t \left(\frac{c_t}{c_{t+1}(\mathbf{s}_{t+1})} \right) \Gamma(\mathbf{s}_t; \mathbf{s}_{t+1}) = q(\mathbf{s}_t; \mathbf{s}_{t+1}), \qquad \text{for all } \mathbf{s}_{t+1}, \tag{11}$$

where $\Gamma(\mathbf{s}_t; \mathbf{s}_{t+1})$ is the (equilibrium) probability of next period aggregate states.

We can now define a competitive general equilibrium. The aggregate states $\mathbf{s}_t$ are given by the credit conditions, ξ_t, and the aggregate stock of bonds, B_t. When necessary, we denote aggregate variables with capital letters to distinguish them from individual variables.

Definition 1 (Recursive Equilibrium)**.** *A recursive competitive equilibrium is defined by a set of functions for (i) workers' policies $h_w(\mathbf{s}_t)$, $c_w(\mathbf{s}_t)$, $b_w(\mathbf{s}_t)$, $a_w(\mathbf{s}_t; \mathbf{s}_{t+1})$; (ii) firms' policies $h(\mathbf{s}_t; b_t)$, $d(\mathbf{s}_t; b_t)$, $b(\mathbf{s}_t; b_t)$; (iii) aggregate prices $w(\mathbf{s}_t)$, $R(\mathbf{s}_t)$, $q(\mathbf{s}_t; \mathbf{s}_{t+1})$; and (iv) probability distribution of aggregate states $\Gamma(\mathbf{s}_t; \mathbf{s}_{t+1})$, such that (i) households' policies satisfy the optimality conditions (9)-(11); (ii) firms discount future dividends by $m_{t+1} = \beta u_c(D_{t+1})/u_c(D_t)$, their policies are optimal and satisfy the Bellman's equation (4); (iii) prices clear the markets for labor, bonds, and contingent claims, that is, $h(\mathbf{s}_t; B_t) = h_w(\mathbf{s}_t)$, $b(\mathbf{s}_t; B_t) = b_w(\mathbf{s}_t)$, $a(\mathbf{s}_t; \mathbf{s}_{t+1}) = 0$ for all $\mathbf{s}_{t+1}$; (iv) the probability distribution of next period aggregate states $\Gamma(\mathbf{s}_t)$ is consistent with the aggregation of individual decisions and the stochastic process for ξ_t.*

To illustrate some of the key properties of the model, we look first at the special case without uncertainty (ξ_t is a constant). In this case the enforcement constraint binds in a steady state.

[9] Since workers are homogeneous within a country, the assumption that they can trade contingent claims is irrelevant in the closed-economy version of the model. The market for contingent claims will play a role later when we consider economies that are financially integrated and, therefore, domestic workers can trade contingent claims with foreign workers.

To see this, consider the first order condition for bonds, equation (10), which in a steady state becomes $\delta R = 1$. Using this condition to eliminate R in (7) and taking into account that in a steady state $\mathbb{E}_t m_{t+1} = \beta$, we get $\mu_t = 1 - \beta/\delta > 0$ (which follows from the assumption $\delta > \beta$).

With uncertainty, however, the enforcement constraint could be binding only occasionally. In particular, it could become binding after a large and unexpected decline in ξ_t. In this event, firms will be forced to cut dividends, inducing a change in the discount factor $\mathbb{E}_t m_{t+1}$. Furthermore, the change in the demand for credit affects the equilibrium interest rate. Using (7) we can see that this affects the multiplier μ_t, which in turn impacts on the demand for labor (equation (8)). Instead, an increase in ξ_t may leave the enforcement constraint nonbinding, without any direct effect on the demand for labor. Thus, the responses to credit shocks could be asymmetric: negative shocks induce large contractions, whereas the impact of positive shocks is moderate.

C Financial Integration

We now consider two symmetric countries, domestic and foreign, with the same preferences and technology as described in the previous section. We will use an asterisk to denote variables pertaining to the foreign country. We continue to assume that workers are unable to purchase shares of domestic and foreign firms. However, under financial integration, (i) investors can purchase shares of both domestic and foreign firms; (ii) firms borrow in a global bond market at a common interest rate R_t; (iii) workers can trade state-contingent claims with foreign workers.[10]

Investors/Firms: Because firms are subject to country-specific shocks, investors gain from diversifying the cross-country ownership of shares. It is easy to show that it is optimal for investors to hold the same quantity of domestic and foreign shares. Thus, domestic and foreign investors have the same consumption, which in turn implies a common stochastic discount factor

$$m_{t+1} = m^*_{t+1} = \frac{\beta u_c\left(\frac{D_{t+1}+D^*_{t+1}}{2}\right)}{u_c\left(\frac{D_t+D^*_t}{2}\right)}.$$

Investors' consumption is the sum of the (aggregate) dividends paid by domestic and foreign firms, $(D_t + D^*_t)/2$. Remember that we denoted by D_t the aggregate dividends to distinguish them from the dividends d_t paid by an individual firm.

[10]This assumption is not crucial for the key results of this paper. However, it will be convenient later in the quantitative analysis when we solve the model numerically.

Besides the common stochastic discount factor, firms continue to solve problem (4) and the first order conditions are given by equations (7) and (8). Let's focus on condition (7), which for convenience we rewrite here for both countries as,

$$\mu_t = 1 - R_t \mathbb{E}_t m_{t+1} = 1 - R_t \mathbb{E}_t m^*_{t+1} = \mu^*_t, \tag{12}$$

Since $\mathbb{E}_t m_{t+1} = \mathbb{E}_t m^*_{t+1}$ (equity market integration) and the interest rate R_t is common (bond market integration), equation (12) implies that the Lagrange multipliers are equal in the two countries, that is, $\mu_t = \mu^*_t$. To get some more intuition for why this is the case, suppose, for example, that $\mu_t > 0$ but $\mu^*_t = 0$ so that $\mu_t = 1 - R_t \mathbb{E}_t m_{t+1} = 1 - R_t \mathbb{E}_t m^*_{t+1} > \mu^*_t = 0$. If that was the case, foreign firms will increase their values by borrowing more (which has a shadow cost of 0) and paying more dividend (which has shadow value of $1 - R_t \mathbb{E}_t m^*_{t+1} > 0$), and they will keep doing do so until the multipliers are equalized.

The equalization of the multipliers implies that the wedges on the demand for labor are equalized in the two countries. In fact, equation (8) is still the optimality condition for the choice of labor in both countries, that is,

$$F_h(h_t) = w_t \left(\frac{1}{1 - \mu_t} \right), \tag{13}$$

$$F_h(h^*_t) = w^*_t \left(\frac{1}{1 - \mu^*_t} \right). \tag{14}$$

As we will see, this property is key for shaping the cross-country impact of a credit shock.

Workers: Although workers are still prevented from purchasing the shares of firms, with capital mobility they can lend to both domestic and foreign firms and they can trade contingent claims with foreign workers. Since the two countries could experience different shocks, the contingent claims will now be traded in equilibrium.

The first order conditions characterizing the workers' decisions are still (9)-(11). Since in equilibrium the prices and probabilities of the contingent claims are the same for domestic and foreign workers, condition (11) implies

$$\frac{c_t}{c^*_t} = \frac{c_{t+1}(\mathbf{s}_{t+1})}{c^*_{t+1}(\mathbf{s}_{t+1})}. \tag{15}$$

Therefore, the ratio of consumption for domestic and foreign workers remains constant over time. We denote this constant ratio by χ.

Aggregate States and Equilibrium: In the economy with integrated financial markets, the aggregate states are $\mathbf{s}_t = (\xi_t, \xi_t^*, B_t, B_t^*, A_t)$, where B_t and B_t^* represent the aggregate financial liabilities of firms, and A_t is the aggregate foreign asset position of the domestic country.

The definition of equilibria is analogous to the one provided for the closed economy with some minor adjustments. In particular, we need to take into account that the bond market is global and there is a common discount factor for domestic and foreign firms.

Denote by $W_t = B_t + B_t^*$ the worldwide wealth of households/workers. This is the sum of bonds issued by domestic firms, B_t, and foreign firms, B_t^*. Because the consumption ratio of domestic and foreign workers is constant at χ and the employment policy of firms does not depend on the individual debt but only on the worldwide debt, the recursive equilibrium can be characterized by the state vector $\mathbf{s}_t = (\xi_t, \xi_t^*, W_t)$. Therefore, the assumption of cross-country risk sharing within workers (with the trade of state-contingent claims) and within investors (with the ownership of foreign shares) allows us to reduce the number of *endogenous* states to only one variable, W_t.

Intuitively, to characterize the firms' policies we do not need to know the distribution of liabilities between domestic and foreign firms. We only need to know the worldwide debt, which is equal to W_t. Since investors own an internationally diversified portfolio of shares, effectively there is only one representative global investor. This is similar to a representative firm with two productive units: one unit located in the domestic country and the other in the foreign country. Since both units have a common owner, it does not matter how the debt is distributed between the two units. What matters for investors is the total debt and the total dividends.[11]

The Effects of a Credit Shock: The next proposition characterizes the real impact of a country-specific credit shock when financial markets are integrated.

Proposition 1. *An unexpected change in ξ_t (domestic credit shock) has the same impact on employment and output of domestic and foreign countries.*

Proof. *We have already shown that the Lagrange multiplier μ_t is equalized across countries. If the ratio of wages in the two countries does not change, the first order conditions imply that all*

[11] This is similar to the problem of a multinational firm that faces demand uncertainty in different countries as studied in Goldberg and Kolstad (1995). There are also some similarities with the problem of a multinational bank with foreign subsidiaries. Cetorelli and Goldberg (2012) provide evidence that multinational banks do reallocate financial resources internally in response to country-specific shocks.

firms choose the same employment. To complete the proof we have to show that the cross-country wage ratio stays constant. Because firms in both countries have the same demand for labor and the ratio of workers' consumption remains constant, the first order condition for the supply of labor, equation (9), implies that the ratio of wages does not change.

Thus, the model generates cross-country co-movement in real economic activities even if shocks are not correlated across countries. However, the model does not generate co-movement in financial flows as a negative credit shock in the domestic country generates a credit crunch only in this country while the foreign country could experience a credit boom.

To understand why a negative credit shock in one country generates a credit boom in the other, consider an initial equilibrium in which the enforcement constraint is not binding in either country. Starting from this equilibrium, suppose that only the domestic economy is hit by a negative credit shock (a reduction in ξ_t but not in ξ_t^*), and this induces binding enforcement constraints in both countries. When ξ_t falls in the domestic country, the shadow value of credit increases in both countries. However, since the constraint has not changed for foreign firms, they will take more credit. In other words, foreign firms increase their borrowing to pay more dividends and offset, partially, the reduction in dividends from firms in the domestic country.

We conclude this section by summarizing the main results obtained so far. In a regime with capital mobility, the model generates a high degree of co-movement in real variables in response to country-specific credit shocks. However, unless these shocks are correlated, the model does not generate co-movement in financial aggregates, which is also a distinguished feature of the 2008 crisis. In the next section we will make credit shocks endogenous providing a theory for time variations in credit tightness and cross-country co-movement in financial aggregates.

III Endogenous Credit Shocks

We now interpret ξ_t and ξ_t^* as the *endogenous* prices at which capital can be sold if a firm defaults and its capital is liquidated.

In case of liquidation, the capital of the firm $\bar{k}$ is perfectly divisible, but before it can be reused it needs to be sold to competitive intermediaries which, after paying a per-unit intermediation cost, can resell it to potential final buyers.[12] If financial markets are integrated, the capital can

[12]Even though households are creditors of firms, when a firm defaults they still need the service of these

be sold to domestic and foreign buyers. Both workers and other firms could buy the liquidated capital and transform it one-to-one in consumption goods. Therefore, the maximum price that buyers are willing to pay for one unit of capital is 1. The structure of the market for liquidated capital is characterized by two assumptions.

Assumption 1. *The per-unit intermediation cost, denoted by $\Phi(N_t)$, is a function of the mass of buyers N_t (workers and firms). The cost is strictly decreasing in $N_t \in [0,2]$ and satisfies $\Phi(1) = 1 - \underline{\xi}$ and $\Phi(2) = 1 - \bar{\xi} \geq 0$.*

Since there is a unit mass of workers and a unit mass of firms, the maximum mass of buyers is 2. Therefore, $N_t \in [0,2]$ is the relevant interval for the function $\Phi(N_t)$. At the heart of the above assumption is the empirical observation by Eisfeldt and Rampini (2006) that secondary markets for capital work less efficiently in bad times. It could be micro-founded with matching frictions and increasing returns in the matching technology, in the spirit of Diamond (1982).[13]

Denote by ξ_t the price paid by intermediaries to purchase one unit of capital from liquidated firms. This is the 'liquidation' price for firms. Furthermore, denote by ξ_t^b the price at which intermediaries resell the purchased capital. Since the intermediation sector is competitive, in equilibrium we have $\xi_t^b = \xi_t + \Phi(N_t)$. Therefore, the price ξ_t^b is always bigger than ξ_t. The difference, however, declines if the number of potential buyers N_t increases. The next assumption provides the conditions for participating in the market for liquidated capital.

Assumption 2. *Buyers can purchase liquidated capital from the intermediaries only if their borrowing constraints are slack.*

This assumption is justified by limited enforcement, as we did for the enforcement constraint (3), and by the assumption that the purchase of capital requires credit (i.e., the firm receives resources only after production). If the enforcement constraint is binding, the firm simply cannot

specialized firms (intermediaries) to liquidate the capital. This assumption is necessary to generate an endogenous (and varying across equilibria) resale price of capital.

[13] A sketch of the matching frictions that could lead to similar properties is as follows. Suppose that an intermediary finds a buyer with probability $1 - \Psi(N)$, where N is the number of potential buyers, and $\Psi'(N) < 0$. If the intermediary does not find a buyer, capital fully depreciates. The intermediary would then maximize $[1 - \Psi(N)]\xi^b k - \xi k$, where ξ^b is the price paid by the buyer and ξ is the price paid by the intermediary to the seller. Competition implies that in equilibrium $[1 - \Psi(N)]\xi^b = \xi$. Furthermore, since buyers can transform one unit of capital into one unit of consumption, $\xi^b = 1$. Assuming that $\Psi'(N) < 0$, i.e., increasing returns to scale in the matching technology, could generate multiple equilibria. The increasing transaction cost assumed in Assumption 1 captures, in reduced form, the increasing returns to scale in the matching technology. We are grateful to an anonymous referee for outlining the matching environment described in this footnote.

get financing to buy additional capital. However, if the constraint is not binding, the firm can increase its borrowing and buy liquidated capital. This is why only buyers with nonbinding borrowing constraints can participate in the market for liquidated capital.

Since workers do not face binding constraints (in fact workers are lenders not borrowers), they can always participate in the market for liquidated capital. Firms, on the other hand, could face binding constraints, in which case they are unable to participate. We will refer to a firm with a nonbinding borrowing constraint as *liquid*.

Lemma 1. *In equilibrium* $\xi_t^b = 1$ *and the liquidation price is* $\xi_t = 1 - \Phi(N_t)$.

Proof. *Since buyers can transform one unit of capital to one unit of consumption goods, the maximum price they are willing to pay is 1. In equilibrium workers always participate while no capital is never sold. Since the demand for liquidated capital is bigger than the supply,* $\xi_t^b = 1$. *Perfect competition implies that intermediaries make zero profits in equilibrium and, therefore,* $\xi_t^b = \xi_t + \Phi(N_t)$. *We have already shown that* $\xi_t^b = 1$ *and, therefore, the zero profit condition implies* $\xi_t = 1 - \Phi(N_t)$.

To clarify the role of liquidity, we should think of a period as divided in two subperiods: beginning-of-period and end-of-period. Operational decisions are made at the beginning of the period while default decisions and potential sales of liquidated capital take place at the end of the period.

1. **Beginning-of-period**: Agents form expectations ξ_t^e for the price at which liquidated capital could be sold at the end of the period. Given the expected price, firms make all operational decisions including the input of labor h_t and the intertemporal debt b_{t+1}, subject to the enforcement constraint

$$\xi_t^e \bar{k} \geq F(h_t) + \frac{b_{t+1}}{R_t}. \tag{16}$$

   The constraint depends on the 'expected' price ξ_t^e since the 'actual' price will be formed at the end of the period. Given the expected price and the firm's choices, the borrowing constraint could be binding or nonbinding. If it is nonbinding, the firm is liquid.

2. **End-of-period**: Firms choose whether to default on the debt b_{t+1} contracted at the beginning of the period and the market for liquidated capital would open if some firms

default. If at this stage all nondefaulting firms are liquid (that is, they did not borrow up to the limit at the beginning of the period and constraint (16) is slack), then there will be a measure 1 of firms capable of purchasing the liquidated capital. This guarantees that the price at which the liquidated capital can be sold is $\xi_t = 1 - \Phi(2) = \overline{\xi}$. Otherwise, the mass of liquid firms (i.e., potential buyers) is 0 and the price will be $\xi_t = 1 - \Phi(1) = \underline{\xi}$. In equilibrium, the expected price at the beginning of the period must be equal to the actual price at the end of the period, that is, $\xi_t = \xi_t^e$.

The fact that the borrowing limit at the beginning of the period depends on the expectation of the liquidation price at the end of the period, which in turn depends on how many firms borrow to the limit, creates the conditions for multiple equilibria.

To see why, suppose that at the beginning of the period all agents expect that the liquidation price is $\xi_t^e = \underline{\xi} = 1 - \Phi(1)$. Since the enforcement constraint (16) is tight, firms may choose to borrow up to the limit. If all firms borrow up to the limit, there will be no liquid firms that can purchase capital of defaulting firms at the end of the period (i.e., N_t would be 1 since only workers can participate). This implies that the transaction cost paid by the intermediary will be high and the liquidation price of capital will be low and equal to $\xi_t = \underline{\xi} = 1 - \Phi(1)$, fulfilling the market expectation. Notice that in this equilibrium there is no incentive for an individual firm to deviate and accumulate more liquidity in order to purchase the capital of defaulting firms. Even if the liquidation price of capital is low, the price that potential buyers have to pay is $\xi_t^b = 1$ (see Lemma 1).

On the other hand, suppose that the expected liquidation price at the beginning of the period is $\xi_t^e = \overline{\xi} = 1 - \Phi(2)$. Because the expected price is high, the enforcement constraint (16) is slack, allowing for a credit capacity that could exceed the borrowing needs of the firm. Thus, firms may choose not to borrow up to the limit. But then, in case a firm defaults at the end of the period, there will be a measure 1 of liquid firms capable of purchasing the liquidated capital in addition to workers. This implies that the transaction cost for intermediaries will be low and the market price will be $\xi_t = \overline{\xi} = 1 - \Phi(2)$.

Remark: Before we move on to characterize the equilibria, it would be helpful to discuss the robustness of our argument for multiplicity. There are two key simplifications in our framework.

The first is that firms are homogeneous and, therefore, there is a representative firm. The second is that default never occurs in equilibrium and capital is never traded. Although informally, we now argue that these two simplifications are not essential to generate multiplicity.

The representative firm assumption implies that in any equilibrium either all firms are constrained (so that the potential buyers are only workers and $N = 1$) or none are constrained (so that the potential buyers are $N = 2$). The logic leading to the multiplicity of equilibria could carry through if we introduce heterogeneous firms. In this case, only a fraction of firms will be constrained in equilibrium but this fraction is endogenous and depends on the liquidation price of capital. When the liquidation price is low, the borrowing constraints are tight implying that more firms are constrained. A large fraction of constrained firms then implies that there are fewer potential buyers in the secondary market and the liquidation price is low. On the other hand, a high liquidation price implies that the borrowing constraints are loose and a smaller number of firms are unconstrained. This implies that more firms are able to participate in the secondary market allowing for a high liquidation price.

The fact that no capital is ever transacted on the secondary market is also not essential for multiplicity. To see this consider an extension of the model in which a fraction of firms exogenously shut down in every period and their capital is sold in the secondary market. Since the actual sale of capital does not alter the measure of potential buyers, it would not alter the equilibrium prices and the possibility of multiple equilibria.

A Financial Autarky

Depending on the beginning-of-period aggregate debt B_t, three cases are possible:

1. The liquidation price is $\underline{\xi}$ with probability 1. This arises for an initial state B_t for which firms choose to borrow up to the limit, independently of the expected price $\xi_t^e \in \{\underline{\xi}, \overline{\xi}\}$.

2. The liquidation price is $\overline{\xi}$ with probability 1. This arises for an initial state B_t for which firms choose to borrow less than the limit, independently of the expected price $\xi_t^e \in \{\underline{\xi}, \overline{\xi}\}$.

3. The liquidation price could be either $\underline{\xi}$ or $\overline{\xi}$. This arises for an initial state B_t for which firms choose to borrow up to the limit when the expected price is $\xi_t^e = \underline{\xi}$, but they do not borrow up to the limit when the expected price is $\xi_t^e = \overline{\xi}$.

The third case is of special interest because it allows for multiple self-fulfilling equilibria.

Denote by $\varepsilon_t \in \{0,1\}$ a sunspot shock. The shock takes the value of zero with probability $\bar{p} \in (0,1)$ and 1 with probability $1-\bar{p}$, and it is serially uncorrelated. When multiple equilibria are possible, the low price equilibrium will be selected if $\varepsilon_t = 0$ while the high price equilibrium will be selected if $\varepsilon_t = 1$. Denoting by $\mathbf{s}_t = (B_t, \varepsilon_t)$ the aggregate states, a competitive equilibrium with endogenous ξ_t can be defined recursively as follows.

Definition 2 (Recursive Equilibria for Given $\bar{p}$)**.** *A recursive competitive equilibrium for given $\bar{p} \in (0,1)$ is defined as a set of functions for: (i) aggregate workers' policies $h_w(\mathbf{s}_t; \xi_t^e)$, $c_w(\mathbf{s}_t; \xi_t^e)$, $b_w(\mathbf{s}_t; \xi_t^e)$; (ii) individual firms' policies $h(\mathbf{s}_t; b_t, \xi_t^e)$, $d(\mathbf{s}_t; b_t, \xi_t^e)$, $b(\mathbf{s}_t; b_t, \xi_t^e)$; (iii) aggregate prices $w(\mathbf{s}_t, \xi_t^e)$, $R(\mathbf{s}_t, \xi_t^e)$, and $\xi(\mathbf{s}_t)$; (iv) aggregate labor H_t, dividends D_t, and debt B_{t+1}; (vi) probability distribution for the next period aggregate states $\Gamma(\mathbf{s}_t, \mathbf{s}_{t+1})$, such that (i) households' policies satisfy the optimality conditions (9)-(11); (ii) firms discount future dividends by $m_{t+1} = \beta u_c(D_{t+1})/u_c(D_t)$, their policies are optimal and satisfy the Bellman's equation (4); (iii) the wage and interest rate clear the labor and credit markets; (iv) the liquidation price is consistent with individual firms' policies and liquidity requirement, that is,*

$$\xi(\mathbf{s}_t) = \begin{cases} \underline{\xi}, & \text{if} \quad \begin{cases} \varepsilon_t = 0 \quad \text{and} \quad F\Big(h(\mathbf{s}_t; B_t, \underline{\xi})\Big) + \frac{b(\mathbf{s}_t; B_t, \underline{\xi})}{R(\mathbf{s}_t; \underline{\xi})} = \underline{\xi}\bar{k} \quad \text{or} \\ \varepsilon_t = 1 \quad \text{and} \quad F\Big(h(\mathbf{s}_t; B_t, \overline{\xi})\Big) + \frac{b(\mathbf{s}_t; B_t, \overline{\xi})}{R(\mathbf{s}_t; \overline{\xi})} = \overline{\xi}\bar{k} \end{cases} \\ \\ \overline{\xi}, & \text{if} \quad \begin{cases} \varepsilon_t = 0 \quad \text{and} \quad F\Big(h(\mathbf{s}_t; B_t, \underline{\xi})\Big) + \frac{b(\mathbf{s}_t; B_t, \underline{\xi})}{R(\mathbf{s}_t; \underline{\xi})} < \underline{\xi}\bar{k} \quad \text{or} \\ \varepsilon_t = 1 \quad \text{and} \quad F\Big(h(\mathbf{s}_t; B_t, \overline{\xi})\Big) + \frac{b(\mathbf{s}_t; B_t, \overline{\xi})}{R(\mathbf{s}_t; \overline{\xi})} < \overline{\xi}\bar{k} \end{cases} \end{cases};$$

(v) expectation of liquidation prices are rational, that is, $\xi_t^e = \xi(\mathbf{s}_t)$; (vi) the probability distribution for next period aggregate states $\Gamma(\mathbf{s}_t, \mathbf{s}_{t+1})$ is consistent with the aggregation of individual decisions and the stochastic process for ε_t. In particular, $H_t = h(\mathbf{s}_t; B_t, \xi_t)$, $D_t = d(\mathbf{s}_t; B_t, \xi_t)$, $B_{t+1} = b(\mathbf{s}_t; B_t, \xi_t)$.

The next proposition establishes the existence of sunspot equilibria, that is, beginning-of-period debt B_t for which the liquidation prices $\underline{\xi}$ and $\overline{\xi}$ could both emerge in equilibrium.

Proposition 2. *Let ε_t be a random variable that takes the value of 0 with probability $\bar{p} \in (0,1)$ and 1 with probability $1-\bar{p}$. If $\overline{\xi} - \underline{\xi}$ is sufficiently large, there exists $\underline{B} < \overline{B}$ such that multiple equilibria exist if and only if $B_t \in [\underline{B}, \overline{B})$. Independently of the initial B_t, the economy will reach the multiplicity region with positive probability.*

Proof. *See Appendix C.*

Figure 5 illustrates informally some of the properties of the model and provides the intuition for the proposition.

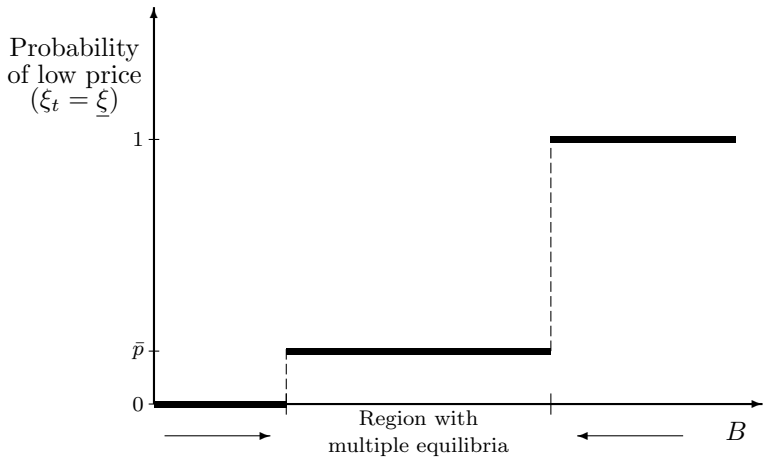

Figure 5: Probability of low price equilibrium and dynamics of debt in autarky.

The probability of a low price equilibrium can take three values depending on the debt. For low values of B_t, the probability of a low price is zero, meaning that the equilibrium is unique and characterized by the high price $\xi_t = \overline{\xi}$. This is because, even if the expected liquidation price is $\xi_t^e = \underline{\xi}$, firms do not borrow up to the limit, that is, $F(h_t) + B_{t+1}/R_t < \underline{\xi}\bar{k}$. Why would firms borrow less than the limit? Given the low value of B_t, choosing a high value of B_{t+1} would imply a large payment of dividends to shareholders. This is not optimal in aggregate since shareholders have concave utility. But then, if firms do not borrow up to the limit, the expectation of a low liquidation price $\xi_t^e = \underline{\xi}$ is not rational. This implies that, independently of the realization of the sunspot shock ε_t, the equilibrium is unique and the price is $\xi_t = \overline{\xi}$.

When the initial debt B_t is large, the equilibrium is also unique and characterized by the low liquidation price $\xi_t = \underline{\xi}$. Thus, the probability of a low price is 1. To see why this is the only possible equilibrium, suppose that the expected liquidation price is $\xi_t^e = \overline{\xi}$ with associated borrowing limit $\overline{\xi}\bar{k}$. Even if firms borrow up to the limit, the high initial debt B_t implies that investors receive small dividend payments. Given the concavity of the utility function, firms would like to pay more dividends, but this requires additional borrowing which pushes them to the limit $\overline{\xi}\bar{k}$. Since firms are constrained, the expectation of a high liquidation price is not rational. Therefore, multiple equilibria can exist only for intermediate values of B_t.

Next we describe why, starting from the two extreme regions where the equilibrium is unique, the economy will move to the intermediate region with multiple equilibria. If we start with a low value of B_t and the borrowing limit is not currently binding, firms have an incentive to increase the stock of debt over time because the discount factor of investors is smaller than the discount factor of workers, that is, $\beta < \delta$. Eventually, they will reach the region with multiple equilibria. On the other hand, if B_t is initially very high and firms are constrained, their input of labor will be inefficiently low. Thus, even if the higher discounting of investors creates an incentive for firms to borrow more, this is counterbalanced by the fact that higher debt must be associated to lower labor (remember that the enforcement constraint is $F(h_t) + B_{t+1}/R_t = \underline{\xi}\bar{k}$). Because of this, firms will reduce their debt and move to the region with multiple equilibria.[14]

B Financial Integration

We will show below that multiple equilibria are also possible when countries are financially integrated. Therefore, sunspot shocks continue to play a role in selecting one of the equilibria in the multiplicity region. It becomes then important whether each country draws its own sunspot shock or there is a single draw of a global sunspot shock. We opt for the first case, as made precise by the following assumption.

Assumption 3. *Each country draws its own sunspot shock independently from each other.*

The assumption imposes that domestic and foreign draws of sunspot shocks, ε_t and ε_t^*, are not correlated across countries. Although in a very stylized fashion, this captures the idea that coordination becomes more difficult when the number of agents increases (an integrated economy is essentially an economy with a larger number of agents). Essentially, we generate this by assuming that agents are not perfectly coordinated across borders. As we will see, this assumption plays an important role for the characterization of equilibria with financially integrated countries.

A central property of the model with financial integration is that the liquidation prices ξ_t and ξ_t^* are equalized across countries. This is stated formally in the following lemma.

[14]Notice that firms are atomistic and, therefore, recognize that multiplicity depends on the aggregate debt B_t, not their individual debt b_t. This also implies that the expected price of liquidated capital and the price of debt is pinned down regardless of the action of an individual firm. The dependence of sunspot equilibria from the aggregate stock of debt is also a feature of the model studied in Cole and Kehoe (2000).

Lemma 2. *With integrated financial markets $\xi_t = \xi_t^*$.*

Proof. *Suppose that the equilibrium is characterized by $\xi_t = \underline{\xi}$ and $\xi_t^* = \overline{\xi}$. To have $\xi_t = \underline{\xi}$ we need $\mu_t > 0$, and to have $\xi_t^* = \overline{\xi}$ we need $\mu_t^* = 0$. However, in Section II C we have shown that with integrated financial markets, $\mu_t = \mu_t^*$. Using the same argument, we can exclude equilibria with $\xi_t = \overline{\xi}$ and $\xi_t^* = \underline{\xi}$.*

Hence, financial integration implies perfect cross-country co-movement in the liquidation prices ξ_t and ξ_t^*. This in turn implies co-movement in financial flows across countries.

The definition of equilibria is similar to the definition provided for the autarky regime with some small adjustments. In particular, the aggregate states are $\mathbf{s}_t = (W_t, \varepsilon_t, \varepsilon_t^*)$ and individual policies are now functions of the expected prices in both countries, that is, ξ_t^e and ξ_t^{e*}. For example, the borrowing policy of domestic firms is denoted by $b(\mathbf{s}_t; b_t, \xi_t^e, \xi_t^{e*})$. Even if in equilibrium the liquidation prices are equalized across countries, this notation allows us to consider out of equilibrium prices and verify what type of expectations induce binding borrowing constraints.

When countries are integrated, it is as if there is only a representative firm that produces and borrows in both countries. This 'globalized' firm faces the consolidated borrowing constraint

$$\xi_t \bar{k} + \xi_t^* \bar{k} \geq F(h_t) + F(h_t^*) + \frac{b_{t+1} + b_{t+1}^*}{R_t}. \tag{17}$$

The left-hand side is the sum of the borrowing limits faced by the globalized firm in each of the two countries. The right-hand side is the total debt given by the intraperiod loan used to finance production in both countries plus the intertemporal debt. The globalized firm does not care in which country it borrows. It only cares about the total debt $b_{t+1} + b_{t+1}^*$.

Since $\xi_t = \xi_t^*$ and changes in these prices are the only source of stochastic fluctuations, we might conclude that financial integration does not change the equilibrium property of the economy. This conclusion, however, is incorrect. Because Assumption 3 imposes that sunspot shocks are uncorrelated across borders, even if liquidation prices are equalized across countries, the probability distribution of $\xi_t = \xi_t^*$ changes when financial markets are integrated.

Assumption 3 is crucial here: countries continue to draw their own sunspot shock independently from each other: ε_t in the domestic country and ε_t^* in the foreign country. Therefore, there are four combinations of realizations of ε_t and ε_t^*. This implies that the probability of a crisis depends on what combinations of ε_t and ε_t^* can induce a low liquidation price $\xi_t = \xi_t^* = \underline{\xi}$.

For example, if a low price equilibrium is possible only if both countries draw a low sunspot shock, that is, $\varepsilon_t = \varepsilon_t^* = 0$, then the probability of a crisis is $\bar{p}^2$. In the autarky regime, instead, this probability is $\bar{p}$ (provided that multiple equilibria are also possible) because only the realization of their own sunspot variable matters when financial markets are not integrated. Alternatively, in the regime with capital mobility we could be in a state in which a negative draw of the sunspot variable in only one of the two countries is sufficient to induce a low price equilibrium. In this case the probability of a crisis is $\bar{p}+2(1-\bar{p})$ while the probability in autarky remains $\bar{p}$ (again, provided that multiple equilibria are also possible).

Proposition 3. *Let ε_t and ε_t^* be two independent random variables that take the value of 0 with probability $\bar{p} \in (0,1)$ and 1 with probability $1-\bar{p}$ in each of the two countries. If $\overline{\xi} - \underline{\xi}$ is sufficiently large, there exist $\underline{B}^A < \overline{B}^A$ and $\underline{B}^M < \hat{B} < \overline{B}^M$ such that*

- *In autarky, multiple equilibria exist if and only if $B_t \in [\underline{B}^A, \overline{B}^A)$. In this region, the probability of a low price equilibrium is $\bar{p}$.*
- *With mobility, multiple equilibria exist if and only if $B_t \in [\underline{B}^M, \overline{B}^M)$. In this region, the probability of a low price equilibrium is $\bar{p}^2$ for $B_t \in [\underline{B}^M, \hat{B})$ and $2\bar{p}-\bar{p}^2$ for $B_t \in [\hat{B}, \overline{B}^M)$.*
- *Let $f^A(B_t)$ and $f^M(B_t)$ be the functions that return B_{t+1}/R_t in autarky and mobility when the liquidation price is high. A sufficient condition for $\underline{B}^M < \underline{B}^A$ is that $f^A(\underline{B}^A) < f^M(\underline{B}^A)$.*

Proof. *See Appendix D.*

The first part of the proposition simply restates Proposition 2. The second part establishes a similar result for the economy with financial integration. Notice that, with some abuse of notation, in the regime with capital mobility we have used B_t to denote the per-country aggregate debt, that is, $B_t = W_t/2$. When multiple equilibria exist, the probability of a crisis differs between the two regimes. In particular, while in autarky the probability is $\bar{p}$, with mobility the probability is initially smaller, $\bar{p}^2$, but then it increases to $2\bar{p} - \bar{p}^2$. The third part of the proposition establishes that $\underline{B}^M < \underline{B}^A$. This implies that in the regime with capital mobility, crises could emerge for smaller values of debt. We established this property under the sufficient condition that for $B_t = \underline{B}^A$, unconstrained borrowing is smaller in autarky.[15]

[15] Although we cannot prove this condition analytically, we believe that this is a general property and, as we will see, it is satisfied in the calibrated model. The reason borrowing should be lower in autarky is because, in the right-hand side neighbor of $\underline{B}^A$, the probability of a crisis is higher in autarky ($\bar{p}$ versus $\bar{p}^2$). Since crises are costly for firms and the cost increases with the debt, a higher probability should discourage firms from borrowing.

Figure 6 provides a graphical illustration of the proposition and clarifies how the regime of capital mobility affects the probability of crises. The two lines show the probability of low price equilibrium (crisis) in the autarky regime (solid line) and under capital mobility (dashed line).

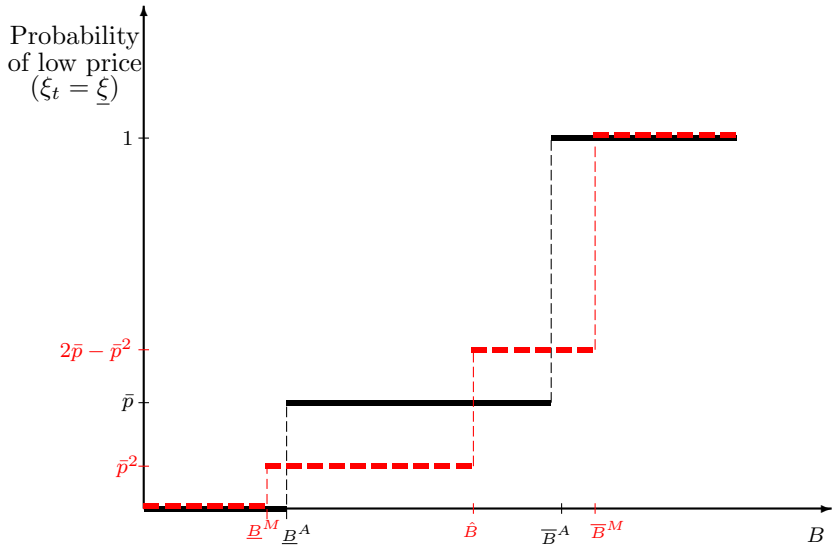

Figure 6: Probability of crises in autarky (solid line) and mobility (dashed line).

Consider first the region where debt is between $\underline{B}^A$ and $\hat{B}$ and the dashed line is below the solid line. In this region, for a given country, the probability of a financial crisis is lower under financial openness than under autarky. Under autarky a crisis happens if the sunspot hits, i.e., it occurs with probability $\bar{p}$. Under mobility if the sunspot hits only one country, because debt is relatively low, this is not enough to trigger the crisis as there is enough borrowing capacity in the other country, so that the borrowing constraint of the global firm is nonbinding. A crisis only hits if the sunspot hits both countries at the same time, i.e., with probability $\bar{p}^2$. Notice thus that in this region financial openness, in a sense, helps countries share the risk of a financial crisis. This desirable feature of openness though induces more borrowing by firms, and that explains the region between $\underline{B}^M$ and $\underline{B}^A$. This is a region where under openness a crisis can happen (with probability $\bar{p}^2$) while in autarky it can't happen: the reason is that under openness firms borrow more, because of the lower risk faced in the $\hat{B}$ and $\underline{B}^A$ region, and that extra borrowing increases the probability of a crisis.

Once the debt is sufficiently large, in particular larger than $\hat{B}$, the contraction of credit in

only one country is sufficient to make the globalized firm constrained. In this case a crisis arises when both countries draw a low realization of the sunspot shock, and also when $\varepsilon_t = 0$ and $\varepsilon_t^* = 1$ or vice versa. Thus, the probability of a crisis becomes $\bar{p}^2 + 2\bar{p}(1-\bar{p}) = 2\bar{p} - \bar{p}^2$. This is bigger than the probability in autarky $\bar{p}$. In this region openness does not provide risk sharing but, in a sense, opens the door to the possibility of financial contagion, as a sunspot in just one country is enough to trigger a global financial crisis.

This figure suggests that the relation between financial openness and vulnerability to financial crisis is complex. For certain value of debt, openness reduces the risk of financial crisis while for other values it increases it. So the overall effect of financial openness will depend crucially on the decisions of agents, i.e., on what is the ergodic set of equilibrium debt. This set is hard to evaluate analytically, but we will characterize it numerically in Section IV B.

C Discussion

Before moving to the quantitative evaluation of our model we offer a brief discussion of the key assumptions made so far.

For analytical simplicity we focused on the financial decisions of nonfinancial firms. However, the mechanism analyzed in the paper can also be thought as operating in other sectors of the economy, specifically, financial intermediation and household sectors. In the case of banks, we can think of ξ_t as the liquidation price for their financial investments. When the banking sector is illiquid, the liquidation value of the banks' assets falls, making the whole banking sector illiquid. In the case of households, we can think of ξ_t as the liquidation price of houses. When this price is low, households are financially constrained and this makes the market for houses illiquid. This could generate a macroeconomic crisis through the collapse in real estate investments. We have chosen not to formalize explicitly these additional mechanisms in order to keep the model simple. However, we would like to think of our model as being more general than simply capturing the changing financial conditions in the nonfinancial corporate sector. As we will see, this view will be in part reflected in the quantitative section of the paper when we calibrate the model.

An important channel in our model makes a crisis global is that in the equilibrium with financial integration, the ownership of firms is perfectly diversified across countries. Although in the data equity ownership remains somewhat home-biased, the bias has declined substantially

during the last 15 years as firms became more globalized and institutional investors expanded their ownership of foreign securities. Therefore, we believe that the mechanism proposed in the paper, although stylized, is important for understanding the global feature of the 2008 crisis.

Of course, cross-country equity ownership is not the only mechanism through which financial integration could make a crisis global. Firms, banks and households could hold other types of foreign securities besides equity. For example, German banks may have purchased collateralized debt securities issued in the United States before the 2008 crisis. So when the housing market in the United States contracted, the value of these securities declined, endangering the financial health of these banks. In this way, a shock that originated in the United States could have propagated to other countries even if investors in the United States did not own any equity of German banks. However, we would like to point out that, from an economic point of view—that is, in terms of investment risk—the ownership of these securities has similar implications to equity ownership. In other words, the financial condition of German banks may worsen either because shareholders demand more dividends or because the US investment has fallen in value. Therefore, we do not consider our transmission mechanism alternative to the mechanism based on the ownership of foreign collateral assets but just one way of formalizing similar financial linkages across countries.

IV Quantitative Analysis

In order to bring the theory to the data, we extend the model in two directions: (i) we add productivity shocks and relax the assumption that the input of capital is fixed; (ii) we allow for costly adjustment of labor and endogenous utilization.

Productivity Shocks and Capital Accumulation. Capital accumulation introduces additional state variables that increase the computational complexity of the model. Since the enforcement constraint is only occasionally binding, the model must be solved using global approximation techniques. These techniques become quickly impractical when there is a large number of state variables. To contain the computational complexity, we assume that the production function is linear in the number of workers and the per-worker production takes the

form

$$y_t = z_t \bar{K}_t^{1-\theta} k_t^{\theta} l_t^{\nu},$$

where y_t is per-worker output, z_t is total factor productivity (TFP), $\bar{K}_t$ is the worldwide per-worker capital, k_t is the individual per-worker capital and l_t is the per-worker *effective* input of labor. The aggregate production function is simply $z_t \bar{K}_t^{1-\theta} k_t^{\theta} l_t^{\nu} N_t$, where N_t is the number of workers. The effectiveness of labor will be explained below. It is further assumed that $\theta + \nu < 1$ and TFP follows a first order Markov process.

The two countries have the same population of workers and investors and we normalize the population of workers to 1. Therefore, $\bar{K}_t = (K_t + K_t^*)/2$. Differences in population will be taken into account in the calibration.

The dependence of the production function from the worldwide per-worker stock of capital, $\bar{K}_t$, introduces an externality. The purpose of the externality is to have constant returns in reproducible factors (AK technology) without losing the competitive structure of the model.

The stock of capital evolves according to

$$k_{t+1} = (1-\tau)k_t + \Upsilon\left(\frac{i_t}{k_t}\right) k_t,$$

where i_t is investment, τ is the rate of depreciation, and the function $\Upsilon(.)$ is strictly increasing and concave, capturing adjustment costs in investment. Adjustment costs are standard in international macro models as they avoid excessive reallocation of capital across countries.

The enforcement constraint is

$$x_t + \frac{b_{t+1}}{R_t} \leq \xi_t k_{t+1},$$

which differs from equation (3) in that the stock of capital is not fixed but it is chosen endogenously by the firm. In case of default k_{t+1} will be sold to intermediaries at price ξ_t as in the model without capital accumulation.

Endogenous Utilization of Labor. The *effective* input of labor l_t results from the combination of (measured) hours, h_t, and (unmeasured) utilization or effort, e_t, according to

$$l_t = A(h_t, e_t) \equiv \left[h_t^{\frac{\varrho-1}{\varrho}} + e_t^{\frac{\varrho-1}{\varrho}} \right]^{\frac{\varrho}{\varrho-1}}.$$

The parameter ϱ is the elasticity of substitution between working hours and actual utilization.

Labor utilization is costly because it generates disutility for workers. More specifically, we assume that the utility function of workers takes the same form as in the previous model. However, the argument of the utility function is the sum of (measured) hours worked, h_t, and (unmeasured) utilization, e_t, that is,

$$U(c_t, h_t + e_t) = \ln(c_t) - \alpha \frac{(h_t + e_t)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}.$$

With this specification, the marginal utilization cost of labor for the firm is w_t and the total cost of labor is $(h_t + e_t)w_t$.

So far, the addition of labor utilization is inconsequential for the properties of the model. Since the wage rate is the price for both h_t and e_t, the CES aggregation implies that firms always choose $e_t = h_t$. Thus, we can simply focus on h_t as we did in the previous model. This no longer holds if firms face some rigidities in the choice of working hours h_t. We formalize this idea by assuming that firms incur the convex cost

$$\kappa(h_t - \bar{h})^2 w_t,$$

where $\bar{h}$ is an exogenous target for per-worker hours.[16] This cost makes hours worked h_t less flexible than utilization e_t. As we will see, this feature allows the model to better capture the dynamics of the labor wedge.

Normalization. We can now take advantage of the AK structure of the model and normalize the growing variables by the worldwide per-worker capital $\bar{K}_t$. Using the tilde sign to denote normalized variables—for example, $\tilde{b}_t = b_t/\bar{K}_t$—we rewrite the budget constraint, the law of motion for capital, and the enforcement constraint for a domestic firm as

$$\tilde{b}_t + \tilde{d}_t + \tilde{i}_t = z_t \tilde{k}_t^{\theta} l_t^{\nu} - \tilde{w}_t(h_t + e_t) - \kappa(h_t - \bar{h})^2 \tilde{w}_t + \frac{g_t \tilde{b}_{t+1}}{R_t}, \tag{18}$$

$$g_t \tilde{k}_{t+1} = (1-\tau)\tilde{k}_t + \Upsilon\left(\frac{\tilde{i}_t}{\tilde{k}_t}\right)\tilde{k}_t, \tag{19}$$

$$\xi_t g_t \tilde{k}_{t+1} \geq z_t \tilde{k}_t^{\theta} l_t^{\nu} + \frac{g_t \tilde{b}_{t+1}}{R_t}, \tag{20}$$

[16] The cost is multiplied by the wage so that it grows with the economy. Ideally, we would like to use a more standard adjustment cost, such as $\kappa(h_t - h_{t-1})^2 w_t$. This alternative formulation, however, would introduce an additional state variable, h_{t-1}, which increases the computational complexity of the model.

where $g_t = \bar{K}_{t+1}/\bar{K}_t$ is the gross growth rate of worldwide per-worker capital. For a foreign firm we would have the same constraints with asterisks on country-specific variables.

As in the model without capital accumulation, investors hold an internationally diversified portfolio of shares, and firms use the common discount factor. With a CRRA utility function for investors, the discount factor takes the form $m_{t+1} = \beta[(D_{t+1} + D^*_{t+1})/(D_t + D^*_t)]^{-\sigma}$, where σ is the risk aversion parameter. In normalized form, it can be rewritten as

$$m_{t+1} = g_t^{-\sigma}\beta\left(\frac{\tilde{D}_{t+1} + \tilde{D}^*_{t+1}}{\tilde{D}_t + \tilde{D}^*_t}\right)^{-\sigma} = g_t^{-\sigma}\tilde{m}_{t+1},$$

and the optimization problem solved by a domestic firm becomes

$$\tilde{V}(\tilde{\mathbf{s}}_t; \tilde{k}_t, \tilde{b}_t) = \max_{\tilde{d}_t, h_t, e_t, \tilde{i}_t, \tilde{b}_{t+1}} \left\{ \tilde{d}_t + g_t^{1-\sigma}\mathbb{E}\tilde{m}_{t+1}\tilde{V}(\tilde{\mathbf{s}}_{t+1}; \tilde{k}_{t+1}, \tilde{b}_{t+1}) \right\} \tag{21}$$

$$\text{subject to (18), (19), (20).}$$

The function $\tilde{V}_t = V_t/\bar{K}_t$ is the normalized firm's value and $\tilde{\mathbf{s}}$ the normalized aggregate states.

We can now see the analytical convenience of the AK structure: by rescaling the problem of the firm by $\bar{K}$, we do not need to keep track of the worldwide stock of capital. Then, to characterize the equilibrium, we only need to keep track of two endogenous states: the normalized worldwide per-capita wealth of workers, $\tilde{W}_t = (\tilde{B}_t + \tilde{B}^*_t)/2$, and the normalized per-worker capital of domestic firms $\tilde{K}_t = K_t/\bar{K}_t$. This is stated formally in the next proposition.

Proposition 4. *Denote by $\tilde{W}_t = (\tilde{B}_t + \tilde{B}^*_t)/2$ the normalized worldwide per-capita wealth of workers and by $\tilde{K}_t = K_t/\bar{K}_t$ the normalized per-worker capital of domestic firms. The sufficient state variables for the characterization of the recursive equilibrium are $\mathbf{s}_t = (\tilde{W}_t, \tilde{K}_t, z_t, z^*_t, \varepsilon_t, \varepsilon^*_t)$.*

Proof. *The online appendix derives the (normalized) first order conditions and shows that $\bar{K}$ does not enter any of these conditions. This follows from the linearity of the production function and the homogeneity of degree 1 of the adjustment cost function $\Upsilon(.)$. Therefore, the normalized policies are independent of the worldwide capital. However, the location of worldwide capital (captured by $\tilde{K}_t$) still matters because capital cannot be freely reallocated (due to the capital adjustment cost) in response to country-specific productivity shocks. Keeping track of the per-capita wealth of workers in each country is not necessary because of consumption risk-sharing among workers. Only the worldwide wealth of workers, which corresponds to the worldwide debt of firms, matters. Also, we do not need to keep track of the allocation of debt between home and foreign firms because, differently from physical capital, there are not adjustment costs in changing the stock of debt.*

The property that the Lagrange multipliers and the efficiency wedge $1/(1-\mu_t)$ are equalized across countries also applies to this extended version of the model. Therefore, if the enforcement constraint is binding in one country, it will also be binding in the other country. The definition of equilibria with endogenous ξ_t and ξ_t^* and the existence of sunspot equilibria are similar to the version of the model without capital accumulation. The online appendix lists the conditions that define a dynamic equilibrium and describes the computational procedure. This concludes the description of the most general version of our model. In the reminder of the paper we first describe the calibration procedure and then proceed to analyze its quantitative properties.

A Calibration

We interpret country 1 as representative of the United States and country 2 as representative of the other G7 countries (Canada, France, Germany, Italy, Japan, and the United Kingdom). Since data for working hours for all G7 countries is only available at an annual frequency, we calibrate the model yearly.

Following the tradition in business cycle studies, the business sector is interpreted broadly: firms are the producers of all goods and services used for consumption and investment, including housing services. This implies that the stock of capital should also be interpreted broadly. Similarly, the debt in the model should be interpreted as representing total private debt in the nonfinancial sector, which includes the debt held by both businesses and households. By doing so, the model can be thought as also capturing the dynamics of some macroeconomic variables that are more specific to the household sector such as real estate investments. Of course, even with this broader interpretation, the model does not capture all the financial mechanisms that could have played a role in the macroeconomic collapse of 2008-2009. Nevertheless, the model should not be interpreted as only capturing the financial channel in the corporate sector.

We start with the calibration of the model without labor rigidities and set $\kappa = 0$. This implies that the endogenous utilization of labor does not play a significant role since firms always choose $e_t/h_t = 1$. The calibration of $A(h_t, e_t)$ will be described when we introduce labor rigidities.

The discount factor for investors, β, and the discount factor for workers, δ, are set to target an average return on equity of 7 percent and an average interest rate of 3.7 percent. While the target for the return on equity is standard, the target for the interest rate is greater than the

real return on government bonds. However, what matters in the model is the interest rate that private borrowers pay, which is significantly higher than the real return on government bonds. We then use the prime rate charged by banks on short-term loans to business from the Federal Reserve Board database. Net of CPI inflation, the average real rate over the period 1995-2012 is 3.7 percent. Given that we are interpreting the capital stock broadly, we are implicitly assuming that the return on equity and the interest rate for the nonbusiness sector are the same as in the business sector (from which we derived the calibration targets). Because of risk, the average return on equity and the average interest rate are different from the intertemporal discount rates. Therefore, to pin down β and δ we need an iterative procedure: after setting all other parameters, we guess the values of β and δ, check the average returns on equity and bonds calculated in the simulated data, and then we update these two parameters.

The utilities of investors and workers take the logarithmic form, that is, $u(c_t) = \ln(c_t)$ and $U(c_t, h_t + e_t) = \ln(c_t) - \alpha(h_t + e_t)^{1+1/\eta}/(1 + 1/\eta)$. The parameter η determines the Frisch elasticity of labor supply which we set to 1, a value that is commonly used in macroeconomic studies. The parameter α is set to target an average working hours of 0.3. The mass of investors is $\omega = 0.25$. Since the mass of workers is 1, this implies that the relative mass of investors is 20 percent.

The production function has two parameters, ν and θ, which we calibrate by imposing two targets: an average labor income share of about 0.7 and a return to scale at the firm level of 0.9. The labor income share in the model is equal to $\nu(1 - \mu_t)$, which is not constant since the multiplier μ_t is positive when the enforcement constraint is binding. However, since in the calibrated model the constraint is binding only occasionally, the average labor share is not very different from ν. Therefore, we set $\nu = 0.7$. The return to scale is determined by $\theta + \nu$, which we set to 0.9. This allows us to pin down $\theta = 0.2$. If we interpret the return to scale as deriving from monopolistic competition, a return to scale of 0.9 implies a price mark-up of about 11 percent, which is close to typical calibrations of macro models with monopolistic competition.

Capital evolves according to $k_{t+1} = (1-\tau)k_t + \Upsilon(i_t/k_t)k_t$ with the function $\Upsilon(.)$ specified as

$$\Upsilon\left(\frac{i_t}{k_t}\right) = \frac{\phi_1}{1-\zeta}\left(\frac{i_t}{k_t}\right)^{1-\zeta} + \phi_2.$$

This functional form is widely used in the literature (for example, in Jermann 1998). The

parameters ϕ_1 and ϕ_2 are chosen so that in the deterministic steady state Tobin's q is equal to 1 ($Q_t = 1$) and investment is equal to the depreciation of capital ($I_t = \tau K_t$). The depreciation rate is set to $\tau = 0.06$ and the curvature parameter ζ is chosen so that investment is about 3 times more volatile than output. The parameter is chosen through an iterative procedure where we guess the value of ζ and check whether the investment volatility generated by the model matches the empirical volatility.

A further modification made for calibration purposes is to assume that the intraperiod loan is only a fraction ψ of the cash-mismatch $x_t = w_t h_t + d_t + b_t - b_{t+1}/R_t$. Since we are calibrating the model annually, the assumption that working capital is equal to the annual production of the firm is probably excessive. Therefore, we assume that the intraperiod loan is ψx_t. This changes the enforcement constraint to $\psi x_t + \frac{b_{t+1}}{R_t} \leq \xi_t k_{t+1}$ but it does not change in any ways the qualitative properties of the model. We set $\psi = 0.2$ which is close to the value used by Bianchi and Mendoza (2013).

At this point, we are left with the parameters directly related to the stochastic properties of the shocks. We start with the sunspot shock. We need to calibrate the liquidation prices $\underline{\xi}$ and $\overline{\xi}$ and the probability $\bar{p}$ (likelihood of drawing the sunspot variable $\varepsilon = 0$). As long as the distance between the low and high liquidation prices is not too small, the value of $\overline{\xi}$ is not important. This is because firms will never borrow up to this limit (again, provided that $\overline{\xi}$ is not too close to $\underline{\xi}$). The calibration of the lower bound is based on a particular interpretation of the recent crisis. We think of the recent crisis as characterized by binding constraints. We can then use the enforcement constraint in 2009 satisfied with the equality sign to calibrate $\underline{\xi}$. More specifically, we use the condition

$$\underline{\xi} K^{G7}_{2010} = 0.2\, Y^{G7}_{2009} + \frac{B^{G7}_{2010}}{R_{2009}}, \tag{22}$$

where K^{G7}_{2010}, Y^{G7}_{2009}, and B^{G7}_{2010}/R_{2009} are, respectively, capital, output and private debt for the aggregation of the G7 countries. Notice that we use 2009 instead of 2008 because the model is calibrated annually and the crisis started in the last quarter of 2008 and the effects fully materialized in 2009. A detailed description of the data and how we aggregate national quantities are provided in Appendix E. Given measurements for the three aggregate variables (see Appendix E), the value of $\underline{\xi}$ is found by solving equation (22). The resulting value is

$\underline{\xi} = 0.738$. If we interpret the intermediation cost as deriving from matching frictions (see footnote 13), then the probability that an intermediary meets a buyer in the bad equilibrium is 0.738.

The probability of a low realization of the sunspot variable is set to $\bar{p} = 0.04$. Thus, a crisis is a low probability event that on average arises every 25 years. Admittedly, because of the low frequency of crises, especially in advanced economies, it is difficult to find the precise calibration target from the data. For this reason we will conduct a sensitivity analysis presented in the online appendix. Also for the calibration of $\bar{p}$ we need to use an iterative procedure.

For the productivity shocks we follow the Solow residual approach, that is, we use the empirical series for output, capital, working hours and labor utilization to compute the TFP variable as a residual from the production function. The problem, however, is that labor utilization e_t is not observable. To solve this problem we impute the value of e_t indirectly using the first order conditions of the firm for the optimal choice of h_t and e_t.

Consider the production function $Y_t = z_t \bar{K}_t^{1-\theta} k_t^{\theta} A(h_t, e_t)^{\nu}$, where $\bar{K}_t$ is worldwide per-worker capital and $A(h_t, e_t)$ is the aggregation of hours and utilization that we specified earlier as a CES function. Combining the first order conditions for the choice of h_t and e_t we obtain

$$\frac{A_h(h_t, e_t)}{A_e(h_t, e_t)} = 1 + 2\kappa(h_t - \bar{h}).$$

This determines e_t as a function of h_t. Therefore, using this condition and the empirical series for output, capital, and hours we are able to construct productivity series as residuals from the production function. A more detailed description is provided in Appendix E.

After constructing the sequences of TFP for the United States, z_t, and for the aggregate of the other G7 countries, z_t^*, we use these series to estimate the following $VAR(1)$ model

$$\begin{pmatrix} \log(z_{t+1}) \\ \log(z_{t+1}^*) \end{pmatrix} = \begin{bmatrix} \rho_1 & \rho_{12} \\ \rho_{21} & \rho_2 \end{bmatrix} \begin{pmatrix} \log(z_t) \\ \log(z_t^*) \end{pmatrix} + \begin{pmatrix} \epsilon_{t+1} \\ \epsilon_{t+1}^* \end{pmatrix}.$$

The estimated parameters are reported in Table 1. We will then approximate the stochastic process for TFP with a discrete Markov chain where z_t and z_t^* can each take three values.

Table 1: Parameter values

Discount factor households/workers, δ	0.986
Discount factor investors, β	0.958
Disutility parameter workers, α	2.700
Labor supply elasticity, η	1.000
Production parameter, θ	0.200
Production parameter, ν	0.700
Depreciation rate, τ	0.060
Capital adjustment parameter, ζ	0.250
Working capital parameter, ψ	0.200
Liquidation prices, $\underline{\xi}$ and $\overline{\xi}$	$\begin{bmatrix} 0.738 & 1.000 \end{bmatrix}$
Frequency of low liquidation price, $\bar{p}$	0.04
Estimated VAR matrix, ρ_1, ρ_{12}, ρ_{21}, ρ_2	$\begin{bmatrix} 0.931 & -0.031 \\ -0.043 & 0.696 \end{bmatrix}$
Estimated standard deviation VAR residuals, σ_ε and σ^*_ε	$\begin{bmatrix} 0.014 & 0.010 \end{bmatrix}$
Mass of investors, ω	0.250

B Crises and Financial Integration

Our first set of numerical results concern how international financial integration affects the probability of a crisis, its depth and the welfare of agents. We believe that these findings have normative implications, especially in light of the policy debate on the desirability of capital controls, which has flourished after the 2008 financial crisis.[17]

The top panels of Figure 7 plot the equilibrium level of debt B_{t+1}, as a function of previous period debt the previous period, B_t, with (Mobility) and without financial integration (Autarky), all conditional on the realizations of productivities taking the mean values.[18] The autarky regime is plotted only for the United States since the plot for the G6 countries in the autarky regime is similar. When there are multiple equilibria, the policy functions are plotted for both equilibria. The solid line plots the policy for the equilibrium with nonbinding constraints (good equilibrium) while the dashed lines plots the debt that is chosen in crisis times. In both regimes, the next period debt in the good equilibrium is an increasing function of current debt, while when expectations become pessimistic debt gets drastically reduced. At some point, however, the

[17] See, for example, Lagarde (2016).

[18] To construct these policies we only impose that the realizations of productivities take the mean values. Agents take into account the stochastic nature of productivity. Debt is normalized by the worldwide capital.

policy function for the good equilibrium debt state crosses the 45 degree line. Therefore, even if expectations continue to be optimistic and good times persist, debt stops growing. The crossing of the solid line determines the upper limit of the ergodic set for debt, which is represented by shaded region in the top panels of Figure 7.

Comparing the two regimes we observe that the ergodic set in the regime with capital mobility is larger than the one in the autarky regime, suggesting that countries do accumulate more debt in a regime of capital mobility.

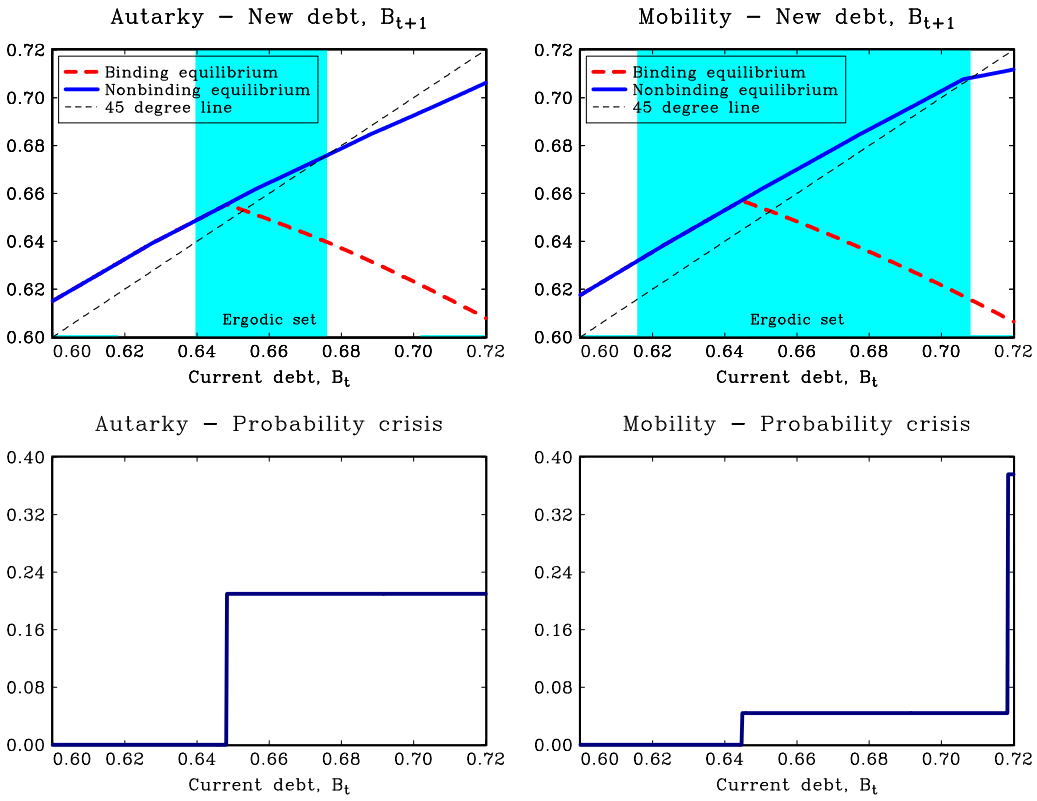

Figure 7: Policy rules, ergodic set, and probability of crises. Autarky regime only for the US.

The reason for this is illustrated by the bottom panels of Figure 7 which plot crisis probabilities and shows, as we discussed in Figure 6, that there exists a large range of values for debt for which this probability is higher in the autarky regime. Higher probability implies higher risk for firms since a crisis forces them to engage in costly deleveraging. This discourages firms from taking on large values of debt. With mobility the probability of a crisis is smaller and this

encourages firms to borrow, and this is the way we observe larger values of debt in equilibrium under capital mobility. Note that for higher values of debt, the crisis probability in the mobility regime becomes higher than the probability in the autarky regime. However, the economy never reaches this point. As the top-right panel in Figure 7 shows, the policy function in good times crosses the 45 degree line before the probability of crisis increases. The upshot of the analysis is that capital mobility reduces the probability of crises but, this reduction in risk induces agents to borrow more and that in turn implies that when crises happen their macroeconomic consequences are bigger.

Table 2 provides a quantitative summary of these results. The statistics reported in the table are computed from the data generated by 1,000 repeated simulations of the model, all starting from the same initial value of debt (the 1995 value in the data). In simulating each version of the model (with and without capital mobility) we use the same random draws of shocks. We report only the statistics for the US economy since the statistics for the G6 countries are similar.

The first line of the table shows that the frequency of financial crises in autarky is significantly higher (18.3 percent versus 4 percent) than under integration. This difference captures the risk sharing benefits of financial integration. On the other hand, the second line shows that under integration there are more global financial crises. Perhaps more concerning, when a crisis arises, the GDP growth of the country experiencing the crisis declines on average by 5.2 percent (relatively to the long-term growth) while the decline in a typical crisis under autarky is only 1.3 percent.

The lower section of Table 2 evaluates the total welfare impact of these two contrasting effects, by measuring the welfare losses, in consumption equivalent units, from closing financial markets. Since there are two types of agents, we calculate welfare losses separately for workers and investors. While workers lose from shutting down financial markets, investors gain. To understand this result observe that reverting to autarky leads firms to borrow less, which in turn leads to lower equilibrium interest rates. Since workers are lenders and investors are borrowers (indirectly through the ownership of firms), lower interest rates are detrimental for workers but beneficial for investors.[19] The last line of the table reports an ex-ante measure of welfare, i.e., the

[19]The finding that financial integration is more beneficial for workers than investors may be at odds with the populist view about the consequences of globalization. However, in relating our welfare finding to the real world there are several considerations that we should take into account. First, our model embeds only one particular

expected loss from reverting to autarky for an agent who does not know whether he/she will be a worker/investor in 1995. In the model workers comprise 80 percent of the population, which is the probability of being a worker. This last line suggests that even though financial integration occasionally leads to larger crisis, a country overall is better off under financial integration.[20]

Table 2: Crises and Integration

	Autarky (%)	*Mobility (%)*
Frequency of crises	18.3	4.0
Frequency of global crises	3.3	4.0
GDP growth slowdown in a crisis	1.3	5.2
Welfare Losses of Going to Autarky (% of lifetime consumption)		
Workers	0.18	
Investors	-0.15	
Ex-ante	0.11	

C International Recessions: Model and Data

In this section we provide the final quantitative evaluation of the model and assess its ability to explain the global crisis of 2008. We simulate the model from 1995 to 2012. We need first to set the initial values of the endogenous states, that is, the normalized worldwide debt, $\tilde{W}_{1995} = \tilde{B}_{1995} + \tilde{B}^*_{1995}$, and the normalized capital of domestic firms $\tilde{K}_{1995} = K_{1995}/\bar{K}_{1995}$. Unless the enforcement constraints are binding, the cross-country distribution of worldwide debt is undetermined in the model. What matters, however, is the total debt. Both variables are initialized using 1995 data.

Given the initial states, to simulate the model we need to construct the sequences of productivity and sunspot shocks for the whole simulation period 1995-2012. The sequence of productivity is constructed using the approach described earlier applied to the United States and the

channel through which financial integration affects the welfare of various agents. In reality, there could be many other channels that affect workers and investors differently. Second, workers are also investors in the model since they buy the bonds issued by firms. Third, workers are homogeneous and, therefore, they are representative of all workers. In particular, they are also representative of high skilled workers who hold significant financial wealth and are sophisticated investors.

[20] Although the welfare losses might appear small, they are significantly larger than the losses found in standard business cycle models without financial frictions as in Cole and Obstfeld (1991).

aggregate of the other G7 countries. The sequence of sunspot shocks, instead, is constructed using our interpretation of the data. Arguably, the 2008-2009 is the only major financial crisis experienced by the G7 countries over 1995-2012 period. Therefore, we assume that until 2008 the draws of the sunspot variables are $\varepsilon_t = 1$. In 2009, however, the draws changed to $\varepsilon_t = 0$ in both countries and returned to $\varepsilon_t = 1$ afterwards. Although the crisis started in the last quarter of 2008, if we consider the whole year, the largest macroeconomic impact took place in 2009. This justifies the choice to locate the crisis in 2009 given that our model is calibrated annually. The sequences of productivity and sunspot shocks are plotted in the top panels of Figure 8.

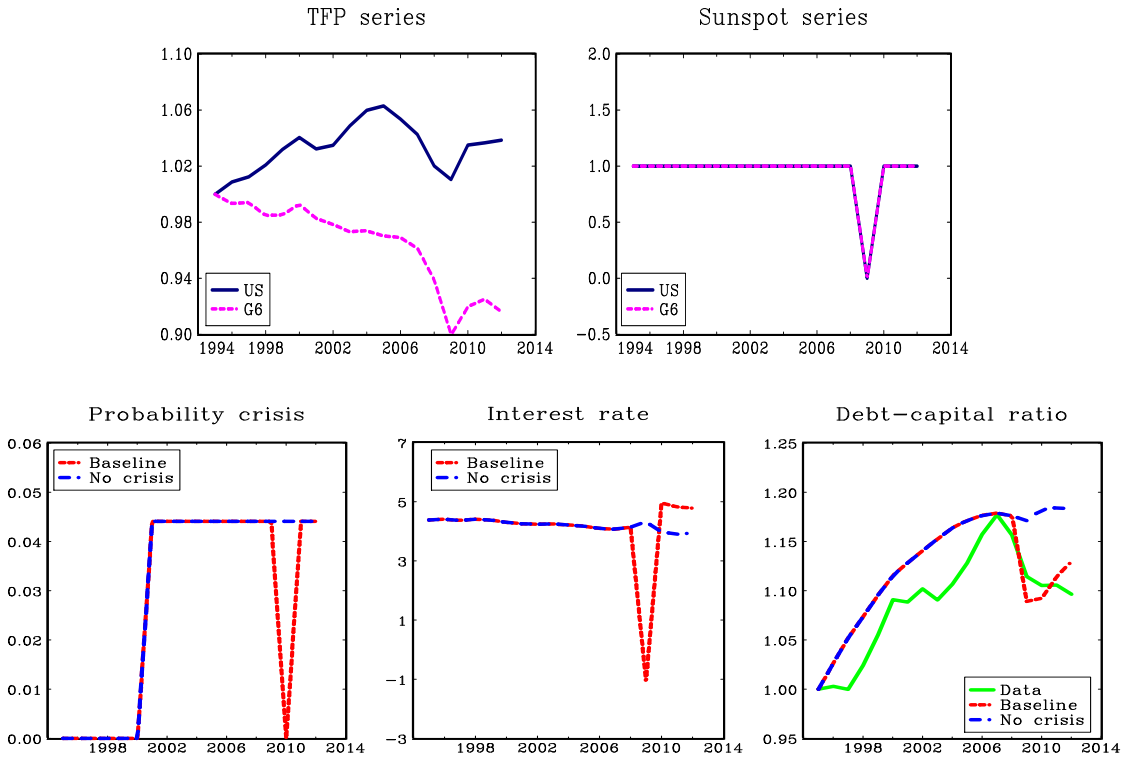

Figure 8: Shocks, probability of crisis, interest rate, and debt.

Two remarks are in order. First, the figure shows that TFP has increased in the US but declined in the G6 countries. The fact that TFP has declined does not mean that GDP has declined in these countries. This is because the model features endogenous growth, and therefore, macroeconomic growth can still be induced by capital accumulation. The second remark is that, the assumption that in 2009 there was the draw of low sunspot shocks does not guarantee a

crisis. For the crisis to materialize, the economy must be in a state in which multiple equilibria are possible. Whether this is the case is indicated by the bottom-left panel of Figure 8. This panel plots the probability of a crisis, that is, the probability of an equilibrium with binding enforcement constraints. As can be seen, this probability was zero in the 1990s but then it became positive (4 percent) in the early 2000s. Therefore, the economy transits from a region where there is a unique equilibrium (and in which sunspot shocks are inconsequential) to a region with multiple equilibria (where sunspot shocks become important for selecting one of the two equilibria). This is due to the the increase in debt. As shown by the bottom right panel of Figure 8, the baseline model generates a significant increase in the stock of debt (smaller dashed-line) until the crisis hits in 2009, which is similar to the data (solid line). Had the negative sunspot shock been drawn in the 1990s, it would have been inconsequential because the debt was not big enough.

Figure 9 plots the responses of working hours, output, and the labor wedge for both the United States and the G6 aggregate, together with the corresponding variables in the data. First we need to describe how we define the labor wedge.

Following the literature, the 'labor wedge' is defined as the difference between the marginal rate of substitution in consumption and leisure and the marginal product of labor. Formally, $U_h/U_c - Y_h$, where U_h and U_c are the marginal utilities of leisure and consumption, respectively, and Y_h is the marginal product of labor. It is customary in the literature to use a CRRA utility in consumption c_t and leisure $1 - h_t$, and a Cobb-Douglas production function in capital k_t and labor h_t. This allows as to write the labor wedge as

$$\text{Labor wedge} = \frac{\phi c_t}{1 - h_t} - (1 - \theta)\frac{y_t}{h_t}, \tag{23}$$

where ϕ is a utility parameter and θ is the capital income share. We use this formula to compute the wedge in the data and in the model (that is, we apply the formula to both real and simulated data). Based on standard calibrations used in the literature, we set $\phi = 1.91$ and $\theta = 0.4$.[21]

Let's focus first on the baseline model (shorter dashed line) and compare it to the data (solid line). The model captures the overall dynamics that preceded the crisis, especially for

[21] Using a standard RBC model with CRRA utility and Cobb-Douglas production, the two parameters are pinned down by targeting steady state hours equal to 0.3, capital income share of 40 percent, and imposing an inter-temporal discount factor of 0.96, a depreciation rate of 0.06, and a steady state growth rate of 2.5 percent.

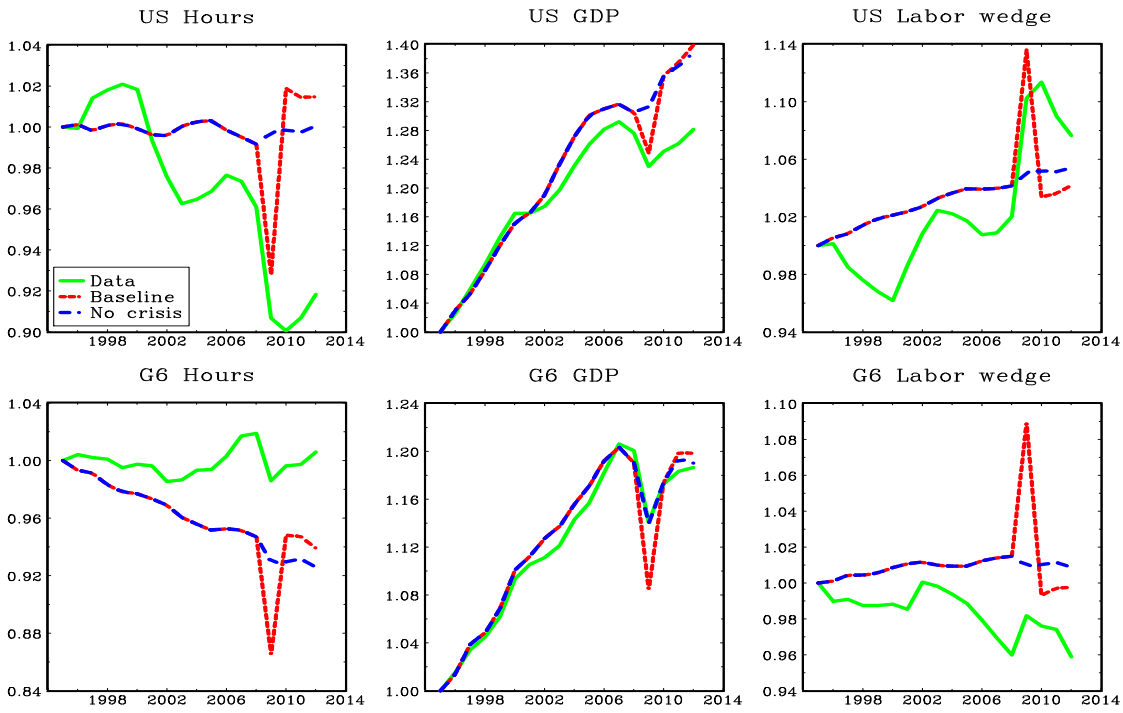

Figure 9: Working hours, output, and labor wedge: the US and the G6 aggregate. Data and model simulation for 1995-2012.

debt (shown in the previous Figure 8), and in GDP. It also captures the 2009 downturn in the US where hours worked and GDP fell and the labor wedge increased by similar magnitudes as in the data. Figure 9 also shows that the credit contraction was crucial for generating the sharp decline in economic activity in 2009. Without the draws of low sunspot shocks (no crisis) the model would not generate the large decline in economic activity (see longer dash line).

The model, however, fails in other dimensions. First, for the G6 aggregate, it predicts a much bigger fall in hours and GDP than in the data. It also predicts a significant increase in the labor wedge, which we do not see in the data for the G6 countries. The second empirical feature that the model fails to capture is the sluggish recovery of the macro-economy.

We conclude that the frictions formalized in our model are important for understanding the built up leading to the crisis as well as the immediate consequences of the crisis, but do not explain why the recovery was slow. Regarding the asymmetry in the response of working hours observed in the United States and in other G7 countries, however, we now show that this could be the result of asymmetries in labor market rigidities.

Labor Markets Rigidities. While the baseline model captures relatively well the US crisis, it overpredicts the response of labor and output in the G6 aggregate. It also predicts a significant increase in the labor wedge of the G6 aggregate which is not present in the data. The heterogeneous response of hours and labor wedge in the US and other G7 countries is well known. Ohanian and Raffo (2012) find that, while the US labor wedge dropped dramatically during the recent crisis, most of the industrialized countries experienced mild declines. At the same time, while the contraction in GDP was not very different, the contraction in hours was much stronger in the US. We will now show that the heterogeneous response of labor can be reconciled, at least in part, with differences in labor market rigidities.

To capture heterogeneous labor market rigidities, we assume that the cost $\kappa(h_t - \bar{h})^2$ could differ between the two countries. We continue to assume that the hiring policy is fully flexible in the United States (that is, $\kappa_{US} = 0$), but it is less so for the G6 aggregate ($\kappa_{G6} > 0$). Although in the baseline model we did not have to calibrate the function $A(h_t, e_t) \equiv \left[h_t^{\frac{\varrho-1}{\varrho}} + e_t^{\frac{\varrho-1}{\varrho}} \right]^{\frac{\varrho}{\varrho-1}}$, with labor frictions the calibration of this function becomes important. We choose a relatively high elasticity of substitution between hours and utilization and set it to $\varrho = 5$. For the employment cost we set $\kappa_{G6} = 1$ and $\bar{h}$ to the average value of working hours which we calibrated to 0.3. The TFP sequence is recalculated using the new parametrization.

Figure 10 compares the simulation of the model with and without hiring frictions. As we can see, adding labor frictions brings the model much closer to the data for the G6 aggregate, without changing the predictions for the United States that were already close to the data. In particular, we now observe that the drop in hours and output in the G6 countries during the crisis is very similar to the data. What the model does not capture is the increase in G6 hours before the crisis. Interestingly, now the model does not generate a significant increase in the labor wedge for the G6 countries. In fact the whole pattern of the labor wedge during the 1995-2012 period is very close to the data.

The stability of the labor wedge during the crisis has a simple intuition. As market conditions deteriorated in the G6 countries, employers reduced utilization (which remained flexible) rather than hours (which were costly to change). This generates a significant drop in production but limited change in hours. For an analyst who looks at the economy through the lens of a standard real business cycle model with full utilization of labor, this will be interpreted as a large drop

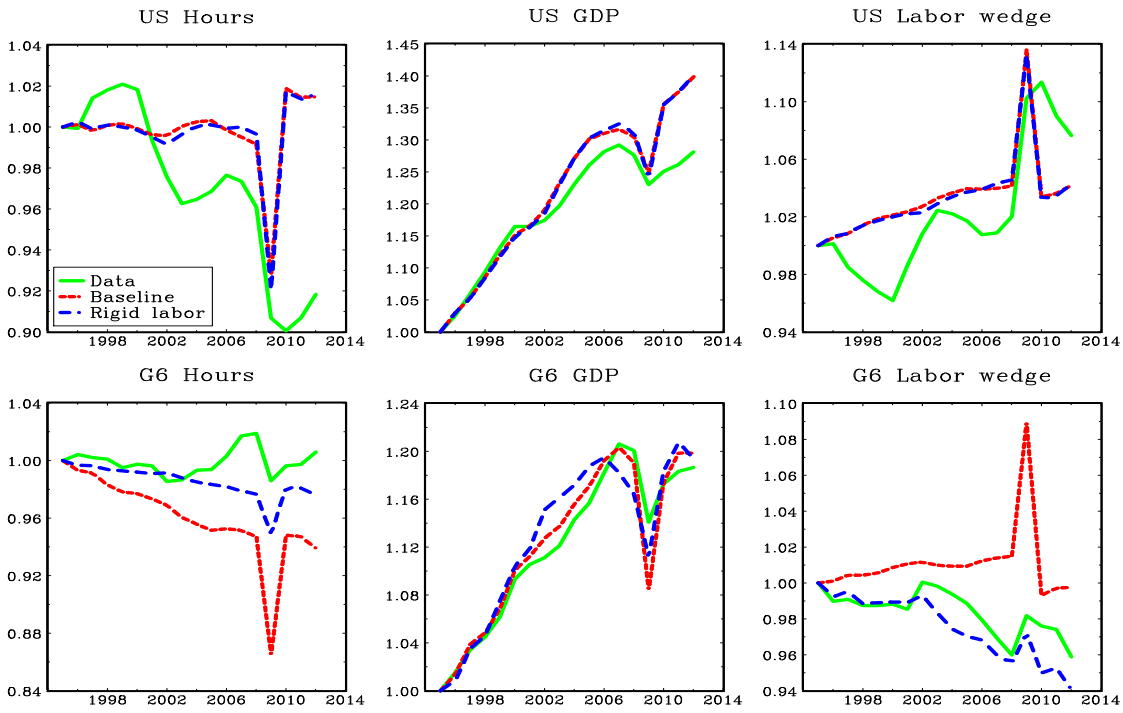

Figure 10: Model with and without hiring rigidities. Data and model simulation for 1995-2012.

in productivity without significant changes in labor efficiency (the labor wedge).

V Conclusion

We have documented that the recent financial crisis has been characterized by a high degree of international synchronization in real and financial variables. We have then proposed a theoretical model that can generate such a co-movement through endogenous credit booms and crises that are generated by self-fulfilling expectations about the liquidity of financial markets. Booms enhance the borrowing capacity of borrowers and lead to higher employment and production. Crises curtail borrowing capacity and lead to sharp contractions in real activity.

Crises are necessarily global when financial markets are integrated. Financial globalization does not only change the degree of financial and macroeconomic synchronization. If also affects the likelihood of crises. When countries are integrated, crises become less frequent. However, because crises are less frequent, borrowers have more incentive to leverage (the precautionary motive becomes less important). As a result, the forced deleveraging induced by a crisis implies

larger macroeconomic contractions.

We have explored the quantitative importance of liquidity induced crises by calibrating the model to the United States and other G7 countries. The simulation over the period 1995-2012 shows that the model captures several features of real and financial data not only during the crisis but also in the period that preceded the crisis. The setup can also help us understand a number of macroeconomic features that are hallmarks of financial crises in general and not specific to the 2008 crisis. An important feature that the model does not capture is the sluggish recovery observed after the crisis. This suggests that although a liquidity shortage is important for explaining the initial collapse in economic activity which is typical of a financial crisis, it cannot explain the sluggish recovery. We think that exploring the potential mechanisms through which a financial crisis can generate a slow recovery is an important direction for future research.

Although this study focuses on the role of financial factors for generating international co-movement, it does not exclude the importance of other channels. As far as the recent crisis is concerned, we do not claim that a credit contraction was the only cause of the crisis. We have shown, however, that credit market conditions can go a long way in capturing some of the salient features of the crisis and, especially, its unprecedented international synchronization. Our findings are also relevant for policy. If crises are driven by global self-fulfilling liquidity shortages, policies geared toward the provision of liquidity in financial markets could avoid bad equilibria and have important real effects. Macro-prudential policies similar to those studied in Bianchi and Mendoza (2013) may also be desirable.

References

Arellano, Cristina, Yan Bai, and Patrick J. Kehoe. 2012. "Financial Frictions and Fluctuations in Volatility." Federal Reserve Bank of Minneapolis Research Department Staff Report 466.

Bacchetta, Philippe, Kenza Benhima, and Céline Poilly. 2014. "Corporate Cash and Employment." Swiss Finance Institute Research Paper No. 14-01. https://ssrn.com/abstract=2394742.

Bacchetta, Philippe, and Eric van Wincoop. 2016. "The Great Recession: A Self-Fulfilling Global Panic." *American Economic Journal: Macroeconomics* 8 (4): 177-98.

Backus, David K., Patrick J. Kehoe, and Finn E. Kydland. 1992. "International Real Business Cycles." *Journal of Political Economy* 100 (4): 745-75.

Benmelech, Efraim, and Nittai K. Bergman. 2012. "Credit Traps." *American Economic Review* 120 (6): 3004-32.

Bentolila, Samuel, Marcel Jansen, Gabriel Jiménez, and Sonia Ruano. 2013. "When Credit Dries Up: Job Losses in the Great Recession." IZA Discussion Paper 7807.

Bianchi, Javier. 2011. "Overborrowing and Systemic Externalities in the Business Cycle." *American Economic Review* 101 (7): 3400-26.

Bianchi, Javier, and Enrique Mendoza. 2013. "Optimal Time-Consistent Macroprudential Policy." NBER Working Paper 19704.

Boissay, Frederic, Fabrice Collard, and Frank Smets. 2013. "Booms and Systemic Banking Crises." ECB Working Paper 1514.

Brunnermeier, Markus, and Yuliy Sannikov. 2014. "A Macroeconomic Model with a Financial Sector." *American Economic Review* 104 (2): 379-421.

Cetorelli, Nicola, and Linda S. Goldberg. 2012. "Banking Globalization and Monetary Transmission." *Journal of Finance* 67 (5): 1811-43.

Chodorow-Reich, Gabriel. 2014. "The Employment Effects of Credit Market Disruptions: Firm level Evidence from the 2008-9 Financial Crisis." *Quarterly Journal of Economics* 129 (1): 1-59.

Christiano, Lawrence J., Roberto Motto, and Massimo Rostagno. 2014. "Risk Shocks." *American Economic Review* 104 (1): 27-65.

Claessens, Stijn, Ayhan Kose, and Marco E. Terrones. 2011. "Financial Cycles: What? How? When?" CEPR Discussion Paper 8379.

Cole, Harold L., and Timothy J. Kehoe. 2000. "Self-Fulfilling Debt Crises." *Review of Economic Studies* 67 (1): 91-116.

Cole, Harold L., and Maurice Obstfeld. 1991. "Commodity Trade and International Risk Sharing: How Much Do Financial Markets Matter?" *Journal of Monetary Economics* 28 (1): 3-24.

Dedola, Luca, and Giovanni Lombardo. 2010. "Financial Frictions, Financial Integration and the International Propagation of Shocks." *Economic Policy* 27 (70): 319-59.

Devereux, Michael B., and Alan Sutherland. 2011. "Evaluating International Financial Integration under Leverage Constraints." *European Economic Review* 55 (3): 427-42.

Devereux, Michael B., and James Yetman. 2010. "Leverage Constraints and the International Transmission of Shocks." *Journal of Money, Credit and Banking* 42 (s1): 71-105.

Diamond, Peter A. 1982. "Aggregate Demand Management in Search Equilibrium." *Journal of Political Economy* 90 (5): 881-94.

Eisfeldt, Andrea L., and Adriano A. Rampini. 2006. "Capital Reallocation and Liquidity." *Journal of Monetary Economics* 53 (3): 369-99.

Gao, Pengjie, and Hayong Yun. 2009. "Commercial Paper, Lines of Credit, and the Real Effects of the Financial Crisis of 2008: Firm-Level Evidence from the Manufacturing Industry." http://hdl.handle.net/1783.1/72569.

Gertler, Mark, and Peter Karadi. 2011. "A Model of Unconventional Monetary Policy." *Journal of Monetary Economics* 58 (1): 17-34.

Goldberg, Jonathan E. 2013. "Credit-Crunch Dynamics with Uninsured Investment Risk." Federal Reserve Board Finance and Economics Discussion Series 2013-31.

Goldberg, Linda S., and Charles D. Kolstad. 1995. "Foreign Direct Investment, Exchange Rate Variability and Demand Uncertainty." *International Economic Review* 36 (4): 855-73.

Greenstone, Michael, Alexandre Mas, and Hoai-Luu Nguyen. 2014. "Do Credit Market Shocks Affect the Real Economy? Quasi-Experimental Evidence from the Great Recession and Normal Economic Times." NBER Working Paper 20704.

Guerrieri, Veronica, and Guido Lorenzoni. 2011. "Credit Crises, Precautionary Savings, and the Liquidity Trap." NBER Working Paper 17583.

Hart, Oliver, and John Moore. 1994. "A Theory of Debt Based on the Inalienability of Human Capital." *Quarterly Journal of Economics* 109 (4): 841-79.

Imbs, Jean. 2010. "The First Global Recession in Decades." *IMF Economic Review* 58 (2): 327-54.

Jermann, Urban J. 1998. "Asset Pricing in Production Economies." *Journal of Monetary Economics* 41 (2): 257-75.

Jermann, Urban, and Vincenzo Quadrini. 2012. "Macroeconomic Effects of Financial Shocks." *American Economic Review* 102 (1): 238-71.

Khan, Aubhik, and Julia K. Thomas. 2013. "Credit Shocks and Aggregate Fluctuations in an Economy with Production Heterogeneity." *Journal of Political Economy* 121 (6): 1055-1107.

Kiyotaki, Nobuhiro, and John Moore. 1997. "Credit Cycles." *Journal of Political Economy* 105 (2): 211-48.

Kocherlakota, Narayana. 2009. "Bursting Bubbles: Consequences and Cures." Paper presented at the Macroeconomic and Policy Challenges Following Financial Meltdowns Conference, hosted by the International Monetary Fund, Washington, DC.

Kollmann, Robert. 2013. "Global Banks, Financial Shocks, and International Business Cycles: Evidence from an Estimated Model." *Journal of Money, Credit and Banking* 45 (s2): 159-95.

Kollmann, Robert, Zeno Enders, and Gernot J. Müller. 2011. "Global Banking and International Business Cycle." *European Economic Review* 55 (3): 407-26.

Lagarde, Christine. 2016. "The Role of Emerging Markets in a New Global Partnership for Growth." Speech delivered at the University of Maryland, College Park, MD. https://www.imf.org/en/News/Articles/2015/09/28/04/53/sp020416.

Liu, Zheng, and Pengfei Wang. 2014. "Credit Constraints and Self-Fulfulling Business Cycles." *American Economic Journal: Macroeconomics* 6 (1): 32-69.

Liu, Zheng, Pengfei Wang, and Tao Zha. 2013. "Land-Price Dynamics and Macroeconomic Fluctuations." *Econometrica* 81 (3): 1147-84.

Lucas, Robert E., Jr., and Nancy L. Stokey. 2011. "Liquidity Crises." Federal Reserve Bank of Minneapolis Economic Policy Paper 11-3.

Martin, Alberto, and Jaume Ventura. 2012. "Economic Growth with Bubbles." *American Economic Review* 102 (6): 3033-58.

Mendoza, Enrique J. 2010. "Sudden Stops, Financial Crises, and Leverage." *American Economic Review* 100 (5): 1941-66.

Miao, Jianjun, and Pengfei Wang. 2017. "Asset Bubbles and Credit Constraints." Unpublished.

Ohanian, Lee E. 2010. "The Economic Crisis from a Neoclassical Perspective." *Journal of Economic Perspectives* 24 (4): 45-66.

Ohanian, Lee E., and Andrea Raffo. 2012. "Aggregate Hours Worked in OECD Countries: New Measurement and Implications for Business Cycles." *Journal of Monetary Economics* 59 (1): 40-56.

Reinhart, Carmen M., and Kenneth S. Rogoff. 2009. *This Time Is Different: Eight Centuries of Financial Folly.* Princeton, NJ: Princeton University Press.

Schularick, Moritz, and Alan M. Taylor. 2012. "Credit Booms Gone Bust: Monetary Policy, Leverage Cycles, and Financial Crises, 1870-2008." *American Economic Review* 102 (2): 1029-61.

Shleifer, Andrei, and Robert W. Vishny. 1992. "Liquidation Values and Debt Capacity: A Market Equilibrium Approach." *Journal of Finance* 47 (4): 1343-66.

Appendix

A Debt Renegotiation

The enforcement constraint is derived from the following assumptions. Default arises at the end of the period before repaying the intratemporal loan $x_t = F(h_t)$. In case of default, the firm retains the liquidity x_t and the lender confiscates the physical capital which will be sold at price ξ_t.

Define the value of the firm recursively as $V_t(b_t) = d_t + \mathbb{E}_t m_{t+1} V_{t+1}(b_{t+1})$, where m_{t+1} is the discount factor, taken as given by an individual firm. Since default takes place at the end of the period, after paying dividends, the value of not defaulting is $\mathbb{E}_t m_{t+1} V_{t+1}(b_{t+1})$.

In the event of default the parties negotiate a repayment τ_t. If they reach an agreement, the firm continues operation and its value is $\mathbb{E}_t m_{t+1} V_{t+1}(b_{t+1}) + x_t - \tau_t$. The firm retains its continuation value $\mathbb{E}_t m_{t+1} V_{t+1}(b_{t+1})$ plus the liquidity x_t net of the bargained repayment τ_t. We are assuming that after reaching an agrement, the firm pays $x_t - \tau_t$ to the shareholders (investors). This is in addition to the dividends d_t paid at the beginning of the period.[22] Without an agreement the firm retains only the divertible liquidity x_t (threat value). The value of an agreement is the difference between the renegotiation value and the threat value, that is,

$$\mathbb{E}_t m_{t+1} V_{t+1}(b_{t+1}) - \tau_t. \tag{A1}$$

Let's now consider the lender. With an agreement the lender gets $\tau_t + b_{t+1}/R_t$. The intertemporal debt is discounted, since it will be repaid next period. Without an agreement the lender receives the liquidation value, $\xi_t \bar{k}$ (threat value). Thus, the net value from renegotiation is

$$\tau_t + \frac{b_{t+1}}{R_t} - \xi_t \bar{k}. \tag{A2}$$

The net surplus is the sum of the values for the firm, (A1), and the lender, (A2),

$$S_t(b_{t+1}) = \mathbb{E}_t m_{t+1} V_{t+1}(b_{t+1}) + \frac{b_{t+1}}{R_t} - \xi_t \bar{k}. \tag{A3}$$

Under the assumption that the firm has all bargaining power, the default value is $x_t + S_t(b_{t+1})$. Incentive compatibility requires that the value of not defaulting is (weakly) bigger than the value of defaulting, that is,

$$\mathbb{E}_t m_{t+1} V_{t+1}(b_{t+1}) \geq x_t + S_t(b_{t+1}).$$

Substituting the definition of the net renegotiation surplus $S_t(b_{t+1})$ and rearranging, we obtain the enforcement constraint $\xi_t \bar{k} = x_t + b_{t+1}/R_t$.

B First Order Conditions for Problem (4)

Let λ_t and μ_t be the Lagrange multipliers for the budget and enforcement constraints. Taking derivatives with respect to d_t, h_t and b_{t+1} we obtain

$$\begin{aligned}
&1 - \lambda_t = 0 \\
&\lambda_t \Big[F_h(h_t) - w_t \Big] - \mu_t F_h(h_t) = 0 \\
&\mathbb{E}_t m_{t+1} V_b(\mathbf{s}_{t+1}; b_{t+1}) + \frac{\lambda_t}{R_t} - \frac{\mu_t}{R_t} = 0.
\end{aligned}$$

[22] We could allow the firm to retain the liquidity $x_t - \tau_t$ and invest it in bonds with gross return R_t. The payment of dividends, however, weakly dominates the investment in bond since $R_t \mathbb{E}_t m_{t+1} \leq 1$ (see equation (7)).

Using the envelope condition $V_b(\mathbf{s}_t; b_t) = -\lambda_t$ to replace the derivative of the value function in the third equation and eliminating λ_t using the first equation, we obtain (7) and (8).

C Proof of Proposition 2

Sunspot equilibria arise when, for a given state B_t, two equilibria are possible: the equilibrium with binding enforcement constraint and low liquidation price $\xi_t = \underline{\xi}$ (tight credit) and the equilibrium with nonbinding enforcement constraint and high liquidation price $\xi_t = \overline{\xi}$ (loose credit). When both equilibria are possible, the actual equilibrium is selected by the random draw of the nonfundamental shock $\varepsilon \in \{0, 1\}$. If for a given state B_t only one equilibrium is possible, the random draw of $\varepsilon \in \{0, 1\}$ is irrelevant. In what follows we show that there are $\underline{B} < \overline{B}$ for which the 'loose credit' equilibrium is unique if $B_t < \underline{B}$; the 'tight credit' equilibrium is unique if $B_t \geq \underline{B}$; for $\underline{B} \leq B_t < \overline{B}$ the 'tight credit' equilibrium arises with probability $\bar{p}$ and the 'loose credit' equilibrium with probability $1 - \bar{p}$. Furthermore, independently of the initial state B_t, the debt will reach the multiplicity region. We show this by contradiction.

- **Loose credit cannot be a steady state.** Suppose that the initial state B_t is such that the enforcement constraint is not initially binding. Furthermore, suppose that the constraint will not be binding in all future periods. Therefore, we start with the assumption that the economy remains in the loose credit equilibrium and the liquidation price is $\xi_t = \overline{\xi}$ for all t. Since the liquidation price remains constant over time, there is no uncertainty and the economy will converge to a steady state. We now show that in this steady state the enforcement constraint must be binding, contradicting the assumption that loose credit can persist forever.

  From the workers' first order condition (10) evaluated at the steady state we have $\delta R = 1$. From the firms' first order condition (7), also evaluated at the steady state, we have $\beta R = 1 - \mu$. Since $\delta > \beta$, these two conditions imply that the multiplier associated with the enforcement constraint μ is strictly positive. Therefore, the enforcement constraint must be binding in a steady state. This contradicts the assumption that loose credit with $\xi_t = \overline{\xi}$ can be a steady state.

- **Tight credit cannot be a steady state.** If equilibria with tight credit persist forever, the economy becomes deterministic and converges to a steady state with $\xi_t = \underline{\xi}$. We show now that if $\overline{\xi} - \underline{\xi}$ is sufficiently large, the expectation of a high liquidation price $\xi_t = \overline{\xi}$ would make the enforcement constraint nonbinding and moves the economy to a loose credit equilibrium. This contradicts the assumption of the existence of a steady state with tight credit.

  Consider the enforcement constraint evaluated at the steady state associated with $\xi_t = \underline{\xi}$

  $$F(H) + \frac{B}{R} = \underline{\xi}\bar{k},$$

  where H, B, and R are steady state labor, debt, and interest rate. Starting from this equilibrium, if the expectation for the liquidation price switches to $\xi_t = \overline{\xi}$, but firms keep $h_t = H$ and $b_{t+1} = B$, the enforcement constraint is no longer binding. Of course, firms will change h_t and b_{t+1} in response. In fact, if all firms do not change their policies, the allocation does not change but $\mu_t = 0$. Then the first order conditions (7) and (8) would not be satisfied. In particular, firms will increase borrowing and in aggregate $B_{t+1} > B$.

  Let's consider the effects of changing B_{t+1}. For the moment we continue to assume that firms keep (inefficiently) the input of labor constant. From the budget constraint of the firm (5) we can see that this implies a higher payment of dividends in the current period. Starting from the next period, however, the dividends will be lower since a higher stock of debt implies higher interest

payments. Therefore, the stochastic discount factor $m_{t+1} = \beta u_c(D_{t+1})/u_c(D_t)$ increases compared to the steady state. Let's look now at the budget constraint of workers, $C_t + B_{t+1}/R_t = B_t + wH$ (the wage does not change since we are keeping the demand of labor H constant). From the budget constraint we can see that workers' consumption declines in the current period (since workers must save more in order to satisfy the higher demand for debt from firms) but they will consume more in the next period when workers receive higher interest payments. From condition (10) we see that this must be associated to an increase in the interest rate R_t.

We now look more closely at the first order condition (7). Since both R_t and m_{t+1} increase compared to the steady state values, the multiplier μ_t must decline. If the new debt satisfies

$$F(H) + \frac{B_{t+1}}{R_t} < \overline{\xi}\bar{k},$$

that is, the enforcement constraint is nonbinding when the market expects a high liquidation price $\xi_t = \overline{\xi}$, then $\mu_t = 0$. For this to be the case, the change in the liquidation price must be large so that the borrowing capacity of the firm increases substantially. This justifies the condition that $\overline{\xi} - \underline{\xi}$ is sufficiently large. Under this condition, the expectation of a high liquidation price is validated ex-post by the fact that the expectation will make the enforcement constraint nonbinding. By assumption, the expectation of $\xi_t = \overline{\xi}$ happens with probability $\bar{p}$.

So far we have kept h_t constant at H. The input of labor also changes but a similar argument applies even if h_t is chosen optimally.

- **Sunspot equilibria for $B_t \in (\underline{B}, \overline{B})$.** We have proved that neither loose credit equilibria with $\xi_t = \overline{\xi}$ nor tight credit equilibria with $\xi_t = \underline{\xi}$ can persist in the long run. This implies that sunspot equilibria in which the economy switches stochastically between tight and loose credit will emerge. We now show that multiple equilibria are possible only if B_t is not too small or not too large.

  Suppose that B_t is very small and the market expects a low liquidation price, $\xi_t^e = \underline{\xi}$. Therefore, firms face the enforcement constraint

$$F(h_t) + \frac{b_{t+1}}{R_t} \leq \underline{\xi}\bar{k}.$$

  Suppose that the policies of the firm, h_t and b_{t+1}, are such that the enforcement constraint is binding, so that $\mu_t > 0$. From the budget constraint of the firm we can see that, a low value of B_t implies high payments of dividends at time t and lower payments at $t+1$ when the beginning of period debt is higher than today. This implies that $m_{t+1} = \beta u_c(D_{t+1})/u_c(D_t)$ will be high. From the budget constraint of workers, instead, we can see that workers' consumption will be low at time t (since they have to save to absorb the increase in the demand for bonds from firms) but will be higher at time $t+1$. Condition (10) then implies that the interest rate R_t will be high.

  Let's look now at the first order condition of firms, equation (7). Since both R_t and m_{t+1} are high, the multiplier μ_t must be small. Furthermore, the lower the initial debt B_t, the lower the multiplier. Since μ_t cannot be negative, if B_t is sufficiently small, μ_t will be zero, that is, the enforcement constraint will not be binding. Therefore, the expectation of a low liquidation price is not rational and the only equilibrium is with loose credit.

  A similar argument shows that for B_t sufficiently large, the enforcement constraint must be binding even if the market expects a high liquidation price. Here the intuition is that, even if the liquidation price is high, debt has a limit. Therefore, if the initial debt is very high, firms need to reduce their dividends while workers will increase their consumption at time t. This implies that the discount factor m_{t+1} and the interest rate R_t will both be low. The first order condition (7) then implies that $\mu_t > 0$, implying that the enforcement constraint is binding. But then the equilibrium with loose credit cannot exist.

To summarize, when the initial state B_t is sufficiently low, only the equilibrium with loose credit exists. However, even if firms do not borrow up to the limit, they increase the debt, that is, $B_{t+1} > B_t$. When the initial state B_t is sufficiently large, only the equilibrium with tight credit exists. However, firms will reduce the debt, that is, $B_{t+1} < B_t$. Eventually, the debt reaches a region with multiple equilibria (provided that $\overline{\xi} - \underline{\xi}$ is sufficiently large).

D Proof of Proposition 3

The first part of the proposition restates Proposition 2 for the autarky regime with proof provided in Appendix C. With financial integration, the sufficient state is the worldwide stock of debt, that is, $W_t \equiv B_t + B_t^*$. Depending on W_t, we have three cases:

1. The liquidation price is $\underline{\xi}$ with probability 1. This arises if we are in a state in which firms borrow up to the limit independently of the expected price.
2. The liquidation price is $\overline{\xi}$ with probability 1. This arises if we are in a state in which firms do not borrow up to the limit independently of the expected price.
3. The liquidation price is $\underline{\xi}$ with some probability smaller than 1. Two cases are possible:

   (a) The liquidation price is $\underline{\xi}$ with probability $\bar{p}^2$. This arises if we are in a state W_t in which firms choose to borrow up to the limit only when the liquidation prices are expected to be low in both countries, that is, $\xi_t^e = \underline{\xi}$ and $\xi_t^{e*} = \underline{\xi}$. This happens when $\varepsilon_t = 0$ and $\varepsilon_t^* = 0$. Since the draw of the sunspot variable in each country is independent, the probability is $\bar{p}^2$.

   (b) The liquidation price is $\underline{\xi}$ with probability $\bar{p}^2 + 2\bar{p}(1-\bar{p})$. This arises if we are in a state W_t in which firms borrow up to the limit even if the liquidation price is expected to be low in only one country. For example, $\xi_t^e = \underline{\xi}$ and $\xi_t^{e*} = \overline{\xi}$, which happens when $\varepsilon_t = 0$ and $\varepsilon_t^* = 1$. The probability of this event is $\bar{p}(1-\bar{p})$. Cross-country symmetry implies that this is also the case if the expectation of a low price arises in the other country. Therefore, the probability that one of the two countries draws $\varepsilon = 0$ is $2\bar{p}(1-\bar{p})$. Of course, if firms choose to borrow up to the limit whenever the price is expected to be low in one country, they will also borrow up to the limit if the price is expected to be low in both countries, that is, $\xi_t^e = \underline{\xi}$ and $\xi_t^{e*} = \underline{\xi}$. The probability of this event is $\bar{p}^2$. Therefore, the probability that firms borrow up to the limit if the liquidation price is low in at least one country is $2\bar{p}(1-\bar{p}) + \bar{p}^2$.

The proof that in the mobility regime the equilibrium is unique for $B_t < \underline{B}^M$ and $B_t \geq \overline{B}^M$ follows the same steps of the proof of Proposition 2. To show that the probability of crises is $\bar{p}^2$ for $B_t \in (\underline{B}^M, \hat{B})$, denote by $B_{t+1}/R_t = f^M(B_t)$ the equilibrium borrowing in the mobility regime assuming that firms do not face the enforcement constraint in the current period (but they will be subject to the enforcement constraint in future periods) when the current debt is B_t. We refer to this policy as the unconstrained equilibrium policy. Remember that in the proposition we define B_t as the average per-country debt, that is, $B_t = W_t/2$. Now consider the states $B_t = \underline{B}^M$. By definition,

$$\underline{\xi}\bar{k} + \underline{\xi}\bar{k} = F(h_t) + F(h_t^*) + 2f^M(\underline{B}^M). \tag{D1}$$

In other words, if globalized firms could choose in the current period the optimal policy without being constrained, they will choose the policy that would make the constraint exactly binding when $\xi_t = \xi_t^* = \underline{\xi}$. Now consider $B_t = \underline{B}^M + \epsilon$, where ϵ is a positive but infinitesimally small number. There are two equilibria at this state since we have already established that multiple equilibria arises when $B_t \in [\underline{B}^M, \overline{B}^M)$.

Because the increase in current debt is very small compared to $\underline{B}^M$, the change in the unconstrained equilibrium policy cannot be very different. But then, if the credit contraction is only in one country, the enforcement constraint will not be binding. Formally,

$$\underline{\xi}\bar{k} + \overline{\xi}\bar{k} > F(h_t) + F(h_t^*) + 2f^M(\underline{B}^M + \epsilon). \tag{D2}$$

This implies that, the expectation of a low liquidation price in only one country is not sufficient to make the consolidated enforcement constraint binding. So it cannot induce a crisis. A crisis can only emerge if the price is expected to be low in both countries and this arises if $\varepsilon_t = 0$ and $\varepsilon_t^* = 0$, that is, both countries draw a low sunspot shock. The probability of this event is $\bar{p}^2$.

As we increase B_t, the unconstrained equilibrium borrowing also increases. Therefore, there must be $\hat{B} > \underline{B}^M$ that satisfies

$$\underline{\xi}\bar{k} + \overline{\xi}\bar{k} = F(h_t) + F(h_t^*) + f^M(\hat{B}). \tag{D3}$$

The borrowing constraint is satisfied with equality if the liquidation price is expected to be low in only one country. This implies that the draw of a low sunspot shock in only one country is sufficient to trigger a crisis. Applying the same logic we can establish that this also applies for $B_t = \hat{B} + \epsilon$. Of course, if the expectation of a low price in only one country is sufficient to make the consolidated enforcement constraint binding, it will also be binding when the price is expected to be low in both countries. Therefore, for $B_t \in [\hat{B}, \overline{B}^M)$, the probability of a crisis is $\bar{p}^2 + 2\bar{p}(1-\bar{p}) = 2\bar{p} - \bar{p}^2$.

Next we want to show that $\underline{B}^M \leq \underline{B}^A$. Consider $B_t = \underline{B}^A$. By definition

$$\underline{\xi}\bar{k} + \underline{\xi}\bar{k} = F(h_t) + F(h_t^*) + 2f^A(\underline{B}^A), \tag{D4}$$

where the function $B_{t+1}/R_t = f^A(B_t)$ defines the equilibrium borrowing in autarky when liquidation prices are high. Condition (D4) implies that, if liquidation prices are high, firms are not constrained when $B_t = \underline{B}^A$. Under the assumption that $f^A(\underline{B}^A) < f^M(\underline{B})$, in the regime with mobility we have

$$\underline{\xi}\bar{k} + \underline{\xi}\bar{k} < F(h_t) + F(h_t^*) + 2f^M(\underline{B}^A), \tag{D5}$$

if h_t and h_t^* are not smaller than the unconstrained equilibrium in autarky. Since borrowing is higher with mobility, workers' consumption must be smaller. But smaller consumption increases the supply of labor as can be seen from (9). Thus, h_t and h_t^* are not smaller with mobility. Condition (D5) then implies that at $B_t = \underline{B}^A$ firms would be constrained if the liquidation prices are low. Therefore, $\underline{B}^M < \underline{B}^A$.

E Data Series and Calibration

The first country in the model represents the United States and the second by the aggregate of other G7 countries. We refer to the other G7 countries as the G6. In order to construct time series for the G6 we need to aggregate national time series for Canada, Germany, France, Italy, Japan, and UK. Furthermore, since in the model debt is only determined at the worldwide level and the production function of each country depends on the worldwide (per-capita) stock of capital, we need to aggregate the debt and capital series for all the G7 countries. Data is from the OECD statistics. Following is the description of how we construct the series used in the calibration and simulation of the model.

TFP: Productivity is derived as a residual from the production function

$$y_t = z_t \bar{K}_t^{1-\theta} k_t^{\theta} A(h_t, e_t)^{\nu},$$

where y_t is per-worker output, $\bar{K}_t$ is worldwide per-worker capital, k_t is per-worker capital, h_t and e_t are working hours and utilization per-worker. The function $A(h_t, e_t) = [h_t^{\frac{\varrho-1}{\varrho}} + e_t^{\frac{\varrho-1}{\varrho}}]^{\frac{\varrho}{\varrho-1}}$ determines the effective labor input by combining per-worker hours, h_t, and utilization, e_t.

If we knew e_t, we could use empirical series for y_t, $\bar{K}_t$, k_t, and h_t to construct series for productivity z_t using the production function. Unfortunately, e_t is not observable. Therefore, we need to derive e_t indirectly using the first order conditions for h_t and e_t from which we obtain

$$\frac{A_h(h_t, e_t)}{A_e(h_t, e_t)} = 1 + 2\kappa(h_t - \bar{h}),$$

which allows us to determine utilization as a function of hours, $e_t = e(h_t)$. We can then write the effective input of labor only as a function of hours, that is, $= A(h_t, e(h_t))$.

Imposing the equilibrium condition $y_t = Y_t$, $k_t = K_t$, and $h_t = H_t$ (in equilibrium, individual variables are equal to aggregate variables) and normalizing by $\bar{K}_t$, the production function can be rewritten as

$$\tilde{Y}_t = z_t \tilde{K}_t^{\theta} A(H_t, e(H_t))^{\nu}.$$

We can then use empirical series for $\tilde{Y}_t$, $\tilde{K}_t$, and H_t to construct the productivity series z_t. We describe next how we construct the empirical measures of $\tilde{Y}_t$, $\tilde{K}_t$, and H_t for both the United States and the G6.

Capital: We construct capital series using the investment series (Gross Capital Formation). The investment series are available at current national prices. For each country i, we convert them in constant PPP dollars using the formula

$$INV_{i,t}^{PPP\$} = \left(\frac{INV_{i,t}^{NAT_CUR}}{GDP_{i,t}^{NAT_CUR}} \right) \times GDP_{i,t}^{PPP\$},$$

where the superscript $PPP\$$ stands for constant US dollars in PPP terms and NAT_CUR stands for current prices in national currency. Once we have the investment series in constant US dollars for each of the seven countries, we construct the G6 and G7 aggregate by adding the national figures.

The stock of capital for $i = \{US, G7, G6\}$ is calculated using the formula

$$CAP_{i,t+1}^{PPP\$} = (1 - \tau) CAP_{i,t}^{PPP\$} + INV_{i,t}^{PPP\$},$$

with the initial stock $CAP_{i,1995}^{PPP\$}$ set to $INV_{i,1995}^{PPP\$}/(\tau + g - 1)$. The depreciation rate is the calibrated value $\tau = 0.06$ and the gross growth rate is set to $g = 1.025$. The capital series can then be used to compute the series

$$\tilde{K}_t = \frac{CAP_{US,t}^{PPP\$}/POP_{US,t}}{CAP_{G7,t}^{PPP\$}/POP_{G7,t}},$$

where $POP_{i,t}$ is the population of country i at date t.

Output: For output we use GDP at constant US dollars in PPP terms ($GDP_{i,t}^{PPP\$}$). The number for the G6 aggregate is obtained by summing the national figures of each country. Then, for $i = \{US, G6\}$ we compute the variable of interest

$$\tilde{Y}_t = \frac{GDP_{i,t}^{PPP\$}/POP_{US,t}}{CAP_{G7,t}^{PPP\$}/POP_{G7,t}}.$$

Hours: The OECD stats database includes annual hours per worker. Since we need a measure of per-capita hours, that is, the total hours of all employed workers divided by population, we need to make some adjustments. First we compute total hours of all employed workers as

$$TOT_HOURS_{i,t} = WORKER_HOURS_{i,t} \times EMPLOYEES_{i,t}.$$

The G6 aggregate for total hours is obtained by summing the national figures of each country. Then, for $i = \{US, G6\}$ we can compute per-capita hours as

$$H_t = \frac{TOT_HOURS_{i,t}}{POP_{i,t}}.$$

Debt: We use Private Sector Debt. OECD stats database provides this variable as a percentage of GDP. The data is available for all G7 countries starting in 1995. In order to compute an aggregate worldwide measure of debt, the national figures need to be expressed in comparable units. So we follow a similar approach as for investment. We first compute national debt at constant US dollars in PPP terms,

$$DEBT_{i,t}^{PPP\$} = DEBT_GDP_{i,t} \times GDP_{i,t}^{PPP\$}.$$

The G7 aggregate is obtained by summing the national figures of all 7 countries. The normalized measure of debt for the G7 countries is obtained as

$$\tilde{B}_t + \tilde{B}_t^* = \tilde{W}_t = \frac{DEBT_{G7,t}^{PPP\$}}{CAP_{G7,t}^{PPP\$}}.$$

Calibration of $\underline{\xi}$ and Initial Simulation Conditions: The 2009 worldwide debt, $\tilde{W}_{2009}$, is used to calibrate the low liquidation price $\underline{\xi}$. By assuming that the enforcement constraint was binding in 2009, we have that the liquidation price was $\xi_{2009} = \underline{\xi}$. This assumption also allows us to write the enforcement constraint as

$$\underline{\xi} = \frac{\tilde{Y}_{2009} + \tilde{Y}_{2009}^*}{g_{2010}} + \frac{\tilde{W}_{2010}}{R_{2009}}.$$

Therefore, using measures of $\tilde{Y}_{2009}$, $\tilde{Y}_{2009}^*$, $\tilde{W}_{2010}$, R_{2009}, and g_{2010} we pin down $\underline{\xi}$.

In order to simulate the model, we need to initialize the endogenous states $\tilde{W}_t$ and $\tilde{K}_t$ in the first simulation year, 1995. We use 1995 measures of $\tilde{W}$ and $\tilde{K}$ calculated as described above.